Dynamic Strength of Materials

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Presentations for Masters Course in Shock Physics,
Institute of Shock Physics, Imperial College, London
March 24, 25, 2011
Crystal grains, dislocations, slip, polycrystal plasticity/ fracturing
TOPICS

I. Dislocation velocity-dependent mechanics
   1. Thermal activation
   2. Dislocation pile-ups
   3. Combined consequences: \( \sigma = \sigma\left\{ (d\varepsilon/dt), T, \ell^{-1/2} \right\} \)
      (i) Ductile-brittle transition/Charpy impact results
      (ii) Plastic instability/shear banding
      (iii) Taylor cylinder impact results

II. Dislocation generation vs. velocity mechanics
    1. Shock front dislocation generations
    2. Copper, ARMCO iron and tantalum results
    3. Shockless isentropic compression experiments (ICEs)
Constitutive Equation Relations

The total range for creep, slip, twinning and cleavage dynamics:

\[ \varepsilon = \varepsilon \{ \Delta t, \sigma, D, T \} \rightarrow \sigma = \sigma \{ (d\varepsilon/dt), T, \ell^{-1/2} \} \]

1. Thermal activation - strain rate analysis, TASRA, \( (d\varepsilon/dt) = (d\varepsilon/dt) \{ T, \tau_{Th} \} \):
   
   Thus \( (\partial \tau_{th}/\partial T)_{ln[d\varepsilon/dt]}(\partial T/\partial [ln[d\varepsilon/dt]]_{\tau Th}(\partial [ln[d\varepsilon/dt]]/\partial \tau_{Th})_{T} = -1.0 \)

   and \( (d\varepsilon/dt) = (d\varepsilon/dt)_{0}\exp\{-G_0 - \int v^* d\tau_{Th}/k_B T\} \), with \( v^* = A^*b \),

   and \( v^* = W_0/\tau_{Th} \) and \( \tau_{Th} = \tau - (\tau_G + k_{Se}\ell^{-1/2}) \).

2. The Hall-Petch microstructural stress intensities, “k”s:
   
   For a circular pile-up; \( n(\tau - \tau_{0\varepsilon}) = m^*\tau_C \) and \( n = 2\alpha(\tau - \tau_{0\varepsilon})\ell/\pi Gb \)

   thus \( \sigma = m_T[(\tau_G + \tau_{Th}) + (\pi m^* Gb \tau_C/2\alpha)^{1/2}\ell^{-1/2}] = \sigma_{0\varepsilon} + k_\varepsilon\ell^{-1/2} \)

   and \( k_{Al} < k_{Cu} < k_{Mg} << k_{\alpha-Fe} \) with \( k_\varepsilon < k_{y.p.} << k_T \sim k_C << K_{IC} = \sigma(\pi c)^{1/2} \)

   with \( c \) and \( \ell \) being analogous in comparison with the fracture mechanics \( K_{IC} \).
“Dynamic Strength of Materials” Bibliography

Zerilli-Armstrong Constitutive Equations

\[
\frac{d\varepsilon}{dt} = \frac{1}{m} \rho b \nu
\]

\[
\nu = \nu_0 \exp\left[-\left(G_0 - \int A^*b d\tau_{Th}/k_B T\right)\right] \text{ and } A^*b = W_0/\tau_{Th}
\]

Computational (Z-A) equations:

\[
\sigma = \sigma_G + B \exp[-\beta T] + B_0\left[\varepsilon_r(1 - \exp\{-\varepsilon/\varepsilon_r\})\right]^{1/2} \exp[-\alpha T] + k_\varepsilon \ell^{-1/2}
\]

in which

\[
(\beta, \alpha) = (\beta_0, \alpha_0) - (\beta_1, \alpha_1) \ln(d\varepsilon/dt)
\]

bcc case: \(\alpha = \alpha_0 = \alpha_1 = 0\)

fcc case: \(B = \beta = \beta_0 = \beta_1 = 0\)

Tensile ductile-brittle transition

\[ \sigma_y = \sigma_C \quad \text{at} \quad T = T_C = \frac{1}{\beta} \left[ \ln B - \ln \left\{ (k_C - k_y) + (\sigma_C - \sigma_0 G) \ell^{1/2} \right\} - \ln \ell^{-1/2} \right] \]

“Impact is the most fundamental of all geological processes,” said Eugene Shoemaker, of the U.S. Geological Survey, who co-discovered the comet that hit Jupiter. “It’s how the Earth was formed.”
Charpy impact transition temperature

\[ \alpha'\sigma_y = \sigma_C \text{ at } T_C = (1/\beta)[\ln\alpha'B - \ln\{(k_C - \alpha'k_y) + (\sigma_{0C} - \alpha'\sigma_{0G})\ell^{1/2}\} - \ln\ell^{-1/2}] \]

Tensile plastic instability: \( \sigma = \frac{d\sigma}{d\varepsilon} \)

\[ \sigma = C + A \varepsilon^n = \frac{d\sigma}{d\varepsilon} = n A \varepsilon^{(n-1)} \]

\[ C/A \varepsilon_u^n \left[ 1 - \left( \frac{n}{n_u} \right) \right] + C/A = 0 \]

bcc case: \( (C/A) = \left( \sigma_0 + B_0 e^{-\beta_T} + k f^{-1/2} \right)/K \)

fcc case: \( n = 1/2, (C/A) = \left( \sigma_0 + k f^{-1/2} \right)/B_1 e^{-\beta_T} \)

BCC vs. FCC plastic instability dependences

Tantalum: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$

Shear banding based on $\sigma = K \varepsilon^n$ and $\sigma = d\sigma/d\varepsilon$; from $(dP/d\ell_0) = 0$ and raised to $(d^2P/d\ell_0^2) = 0$

Shear banding from a dislocation pile-up avalanche

Shear band susceptibility from the pile-up avalanche model

Titanium: \( \sigma = \sigma\{ (d\varepsilon/dt), T, \ell^{-1/2} \} \)

Isothermal and adiabatic stress strain curves for Ti6Al4V material

Copper: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$

Copper Taylor cylinder impact result

Original Taylor cylinder test result on mild steel

SHPB twinning measurements compared to Z-A slip calculations

Armco iron Taylor test involving twinning and slip

Sequential twinning and slip in the Taylor cylinder impact test

Model for a propagating shock front

\[ \sigma = P\{\frac{(1 - \nu)}{(2 - \nu)}\} \] for one-dimensional strain

MD model of shock-induced dislocation dipole structures

Model of 3-D post-shock deformation


Analogy with “channeling” of deformation in post-deformation of neutron-irradiated materials
Transition from strain rate control by dislocation velocity to control by dislocation generation

For dislocation velocity control:

\[
\frac{d\varepsilon}{dt} = \frac{1}{m}\rho bv
\]

\[
\sigma = \sigma_G + B\exp[-\beta T] + B_0[\varepsilon_r(1 - \exp\{-\varepsilon/\varepsilon_r\})]^{1/2}\exp[-\alpha T] + k_c \ell^{-1/2}
\]

in which

\[(\beta, \alpha) = (\beta_0, \alpha_0) - (\beta_1, \alpha_1)\ln(\frac{d\varepsilon}{dt})\]

For dislocation generation control:

\[
\frac{d\varepsilon}{dt} = \frac{1}{m}b(d\rho/dt)\Delta x_d
\]

for which, at limiting small value of \(v^* \sim b^3\), and \(m = 2\)

\[
\sigma_{Th} = (2G_0G/v^*) - (2k_BT/v^*) [\ln\{(d\varepsilon/dt)_0/(d\varepsilon/dt)\}]
\]

SHPB indication of dislocation generation

Connection of Follansbee et al results with Swegle and Grady shock measurements

Grain Size Dependent Slip/Twinning Transition in Iron at the Hugoniot Elastic Limit (HEL)

Hall-Petch Relations:

**Slip**

\[
\sigma = B \exp(-\beta T) + A \varepsilon^n + \sigma_G + k_y \ell^{-1/2}
\]

\[
\beta = \beta_0 - \beta_1 \ln(d\varepsilon/dt)
\]

**Deformation Twinning**

\[
\sigma_T = \sigma_{T0} + k_T \ell^{-1/2}
\]

References

Tantalum

![Graph showing true stress vs. strain rate for Tantalum]

Tantalum Conventional and Shock Results

Ta single crystal/polycrystal SHPB results

Critical activation volumes, $v^*$, under shock loading

<table>
<thead>
<tr>
<th>Metal</th>
<th>$b \times 10^{-10}$ m</th>
<th>$2kT/b^3$ (MPa)</th>
<th>$\Delta\sigma/\Delta\ln[d\varepsilon/dt]$ (MPa)</th>
<th>$V^* (b^3)$**</th>
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<tbody>
<tr>
<td>Cu</td>
<td>2.55</td>
<td>500</td>
<td>505</td>
<td>1.0</td>
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<tr>
<td>Al</td>
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<td>354</td>
<td>252</td>
<td>1.4</td>
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<td>$\alpha$-Fe</td>
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<td>542</td>
<td>1400</td>
<td>0.4</td>
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<tr>
<td>Ta</td>
<td>2.84</td>
<td>361</td>
<td>1200</td>
<td>0.3</td>
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** $V^* = mkT(\partial \ln[d\varepsilon/dt]/\partial \sigma)_T$
Laser-shocked deformation of copper

The strain and strain rate from combined dislocation displacements and generations

The strain from displaced and generated dislocations

\[ \Delta \varepsilon = (1/m)[\rho_N b \Delta x_N + \Delta \rho_G b \Delta x_d + \rho_G b \Delta x_d] \]

The strain rate is obtained then for the presumed time-dependent parameters as

\[ \frac{d\varepsilon}{dt} = (1/m)[\rho_N b \nu_N + (d\rho_G/dt)b \Delta x_d + \rho_G b \nu_G] \]

with neglect of \((d\rho_N/dt)\). If it were assumed that \(\nu_N = \nu_G = \nu^*\), and \(\rho_N\) and \(\rho_G\) could be combined as \(\rho_T\) then

\[ \frac{d\varepsilon}{dt} = (1/m)[(d\rho_G/dt)b \Delta x_d + \rho_T b \nu^*] \]
Shockless isentropic compression experiments (ICEs)

The resident dislocation density is required to “carry the load”, and because $\rho_N$ is low, $\nu_N$ is so high as to be controlled by “drag”!

$$\sigma_{Th} = \{1 - [c(d\varepsilon/dt)/\beta_1 \sigma_{Th}]^{-\beta_1 T}\} [B \exp(-\beta_T)]$$

in which

$$c = c_0 m^2 \beta_1 / \rho b^2 \text{ and } b\tau_{TH} = c_0 \nu.$$ 

At high $(d\varepsilon/dt)$:

$$\sigma_{Th} = (c_0 m^2 / \rho b^2)(d\varepsilon/dt)$$

Copper SHPB, Shock, and ICE results

Drag-controlled shockless ICE results for copper

Strength of Material Dynamics

SUMMARY

1. An introduction has been given on a dislocation mechanics basis to coupling of model thermal activation – strain rate analysis (TASRA) and grain size related, dislocation pile-up, constitutive relations.

2. Attention was directed to the strain rate, temperature, and grain size dependencies that are established for evaluations of such material strength properties as: (i) conventional stress – strain; (ii) ductile-brittle transition; (iii) Charpy v-notch impact; (iv) plastic instability; (v) shear banding; (vi) Taylor cylinder impact; (vii) shock; and, (viii) shockless isentropic compression experiments (ICEs).

3. Beyond giving emphasis to the dislocation velocity and generation being thermally activated processes at the crystal lattice scale, the fuller description involves key features of (i) dislocation pile-up associated internal stress concentrations, (ii) relief needed for dislocation generations at all lattice points along a propagating shock front; and, (iii) trade-offs between dislocation density and velocity in distinguishing between very different strength properties at comparable shock and shockless deformation rates.