Dynamic Strength of Materials

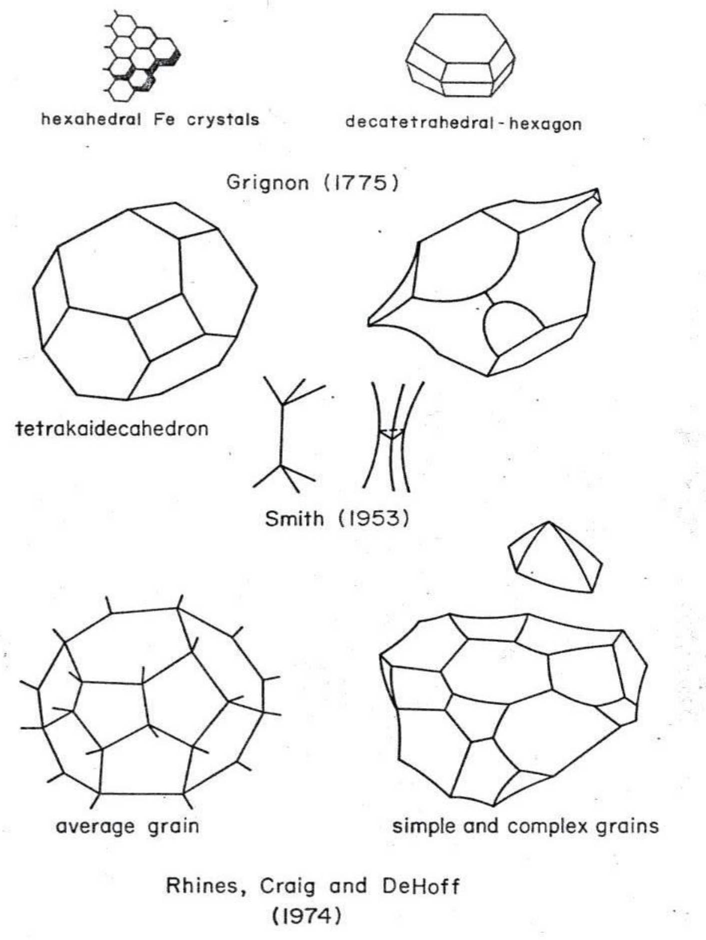
Ronald W. Armstrong

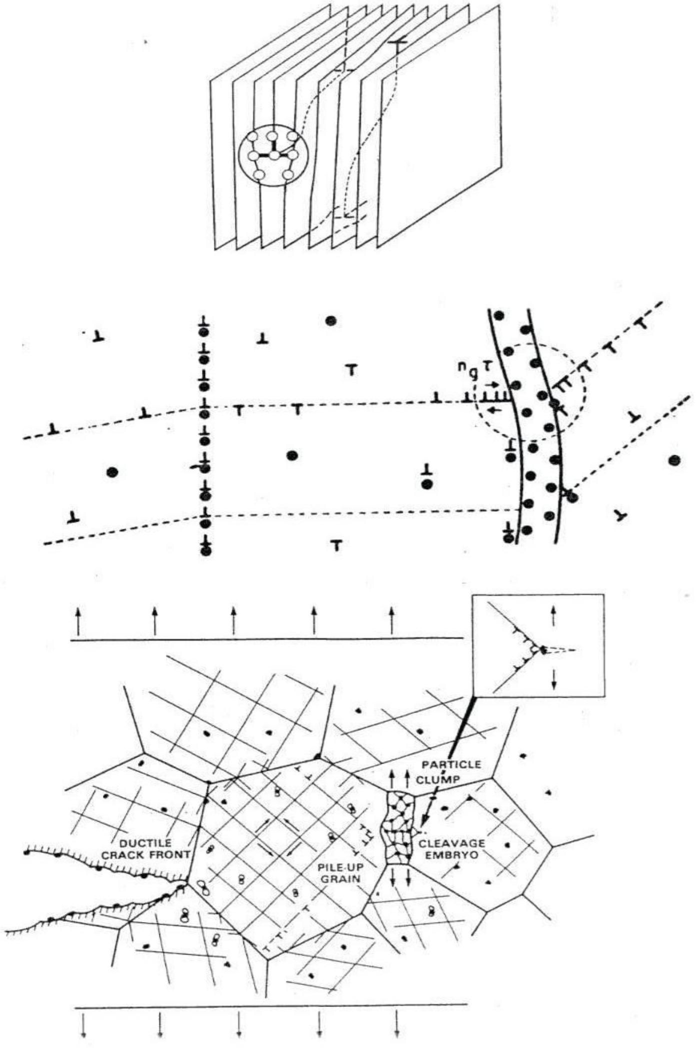
Center for Energetic Concepts Development
Department of Mechanical Engineering
University of Maryland, College Park, MD 20742

Presentations for Masters Course in Shock Physics, Institute of Shock Physics, Imperial College, London

March 24,25, 2011

Crystal grains, dislocations, slip, polycrystal plasticity/ fracturing





TOPICS

I. Dislocation velocity-dependent mechanics

1. Thermal activation
2. Dislocation pile-ups
3. Combined consequences: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$
   (i) Ductile-brittle transition/Charpy impact results
   (ii) Plastic instability/shear banding
   (iii) Taylor cylinder impact results

II. Dislocation generation *vs*. velocity mechanics

1. Shock front dislocation generations
2. Copper, ARMCO iron and tantalum results
3. Shockless isentropic compression experiments (ICEs)

Constitutive Equation Relations

The total range for creep, slip, twinning and cleavage dynamics:

$$\varepsilon = \varepsilon\{\Delta t, \sigma, D, T\} \rightarrow \sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$$

1. Thermal activation - strain rate analysis, TASRA, $(d\varepsilon/dt) = (d\varepsilon/dt)\{T, \tau_{Th}\}$:
   thus $(\partial\tau_{th}/\partial T)_{\mathbf{ln}[d\varepsilon/dt]}(\partial T/\partial \mathbf{ln}[d\varepsilon/dt])_{\tau Th}\ (\partial \mathbf{ln}[d\varepsilon/dt]/\partial\tau_{Th})_T = -1.0$
   and $(d\varepsilon/dt) = (d\varepsilon/dt)_0 \mathbf{exp}\{-(G_0 - \int v^* d\tau_{Th})/k_B T\}$, with $v^* = A^*b$,
   and $v^* = W_0/\tau_{Th}$ and $\tau_{Th} = \tau - (\tau_G + k_{S\varepsilon}\ell^{-1/2})$.

2. The Hall-Petch microstructural stress intensities, “k”s:
   For a circular pile-up; $n(\tau - \tau_{0\varepsilon}) = m^*\tau_C$ and $n = 2\alpha(\tau - \tau_{0\varepsilon})\ell/\pi G b$
   thus $\sigma = m_T[(\tau_G + \tau_{Th}) + (\pi m^* G b \tau_C/2\alpha)^{1/2}\ell^{-1/2}] = \sigma_{0\varepsilon} + k_\varepsilon \ell^{-1/2}$
   and $k_{Al} < k_{Cu} < k_{Mg} << k_{\alpha\text{-}Fe}$ with $k_\varepsilon < k_{y.p.} << k_T \sim k_C << K_{IC} = \sigma(\pi c)^{1/2}$
   with c and ℓ being analogous in comparison with the fracture mechanics K_{IC}

“Dynamic Strength of Materials” Bibliography

1. A. Seeger, in “***Dislocations and Mechanical Properties of Crystals***”, edited by J.C. Fisher, W.G. Johnston, R. Thomson, and T. Vreeland, Jr. (John Wiley & Sons, Inc., N.Y., 1956) p. 243.
2. “***Dislocation Dynamics***”, edited by A.R. Rosenfield, G.T. Hahn, A.L. Bement, Jr., and R.I. Jaffee (McGraw-Hill Book Co., N.Y., 1968).
3. R.W. Armstrong, in “***Yield, Flow and Fracture of Polycrystals***”, edited by T.N. Baker (Appl. Sci. Publ., London, U.K., 1983) p. 1.
4. “***Mechanics of Materials***”, edited by M.A. Meyers, R.W. Armstrong, and H.O.K. Kirchner (John Wiley & Sons, Inc., N.Y., 1999).
5. R.W. Armstrong and S.M. Walley, “High Strain Rate Properties of Metals and Alloys”, ***Intern. Mater. Rev.* 53**, [3], 105-128 (2008).
6. R.W. Armstrong, W. Arnold, and F.J. Zerilli, “Dislocation mechanics of shock-induced plasticity”, ***J. Appl. Phys.,*** 105, 023511 (2009), 7 pp.
7. R.W. Armstrong, “Dislocation Viscoplasticity Aspects of Material Fracturing”, ***Eng. Fract. Mech***., 77, 1348-1359 (2010).
8. R.W. Armstrong, in “**Mechanical Properties of Nanocrystalline Materials**”, edited by J.C.M. Li (Pan Stanford Publ., Singapore, 2011) Chap. 3, pp. 61-91, in print.

Zerilli-Armstrong Constitutive Equations

$$(d\varepsilon/dt) = (1/m)\rho b\nu$$

$$\nu = \nu_0 \mathbf{exp}[-(G_0 - \int A^* b d\tau_{Th})/k_B T] \quad \text{and} \quad A^* b = W_0/\tau_{Th}$$

Computational (Z-A) equations:

$$\sigma = \sigma_G + B\mathbf{exp}[-\beta T] + B_0[\varepsilon_r(1 - \mathbf{exp}\{-\varepsilon/\varepsilon_r\})]^{1/2}\mathbf{exp}[-\alpha T] + k_\varepsilon \ell^{-1/2}$$

in which

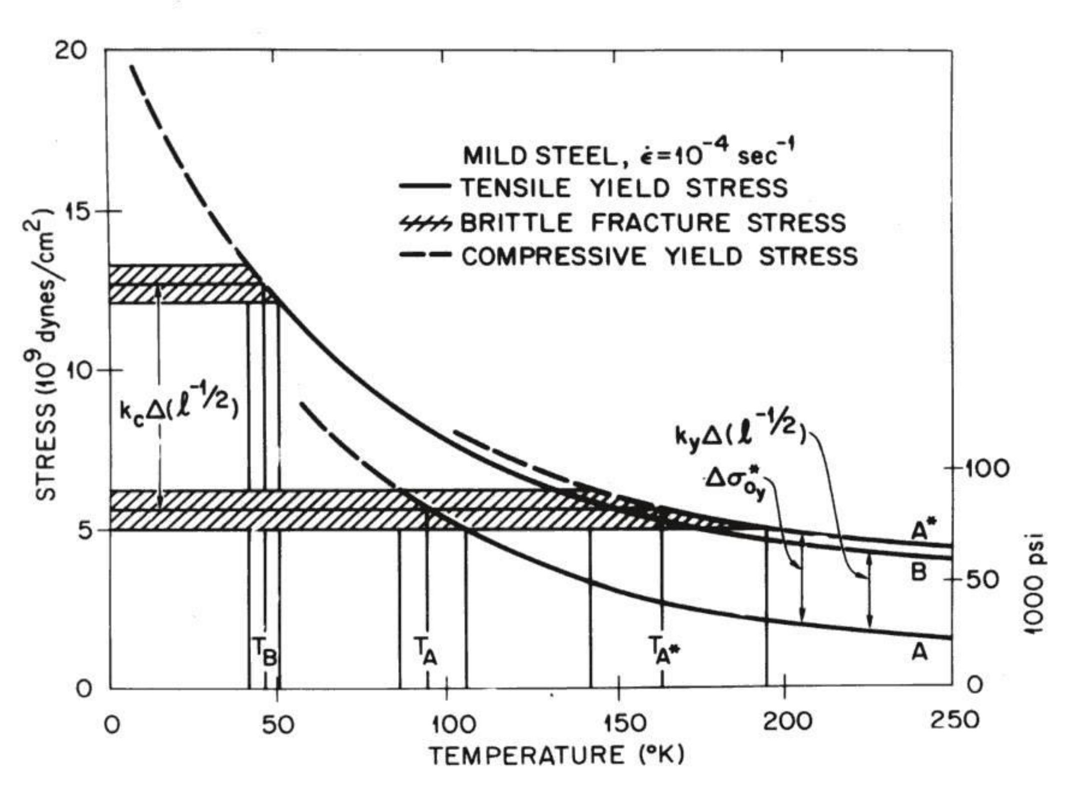
$$(\beta,\alpha) = (\beta_0,\alpha_0) - (\beta_1,\alpha_1)\mathbf{ln}(d\varepsilon/dt)$$

bcc case: $\alpha = \alpha_0 = \alpha_1 = 0$

fcc case: $B = \beta = \beta_0 = \beta_1 = 0$

F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.* **61**, 1816-1825 (1987)
F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.* **68**, 1580-1591 (1990)
F.J. Zerilli, *Metall. Mater. Trans. A*, **35A**, 2547-2555 (2004)

Tensile ductile-brittle transition



$\sigma_y = \sigma_C$ at $T = T_C = (1/\beta)[\mathbf{ln}B - \mathbf{ln}\{(k_C - k_y) + (\sigma_C - \sigma_{0G})\ell^{1/2}\} - \mathbf{ln}\ell^{-1/2}]$

R.W. Armstrong, *Metall. Mater. Trans.*, **1**, 1169-1176 (1970).

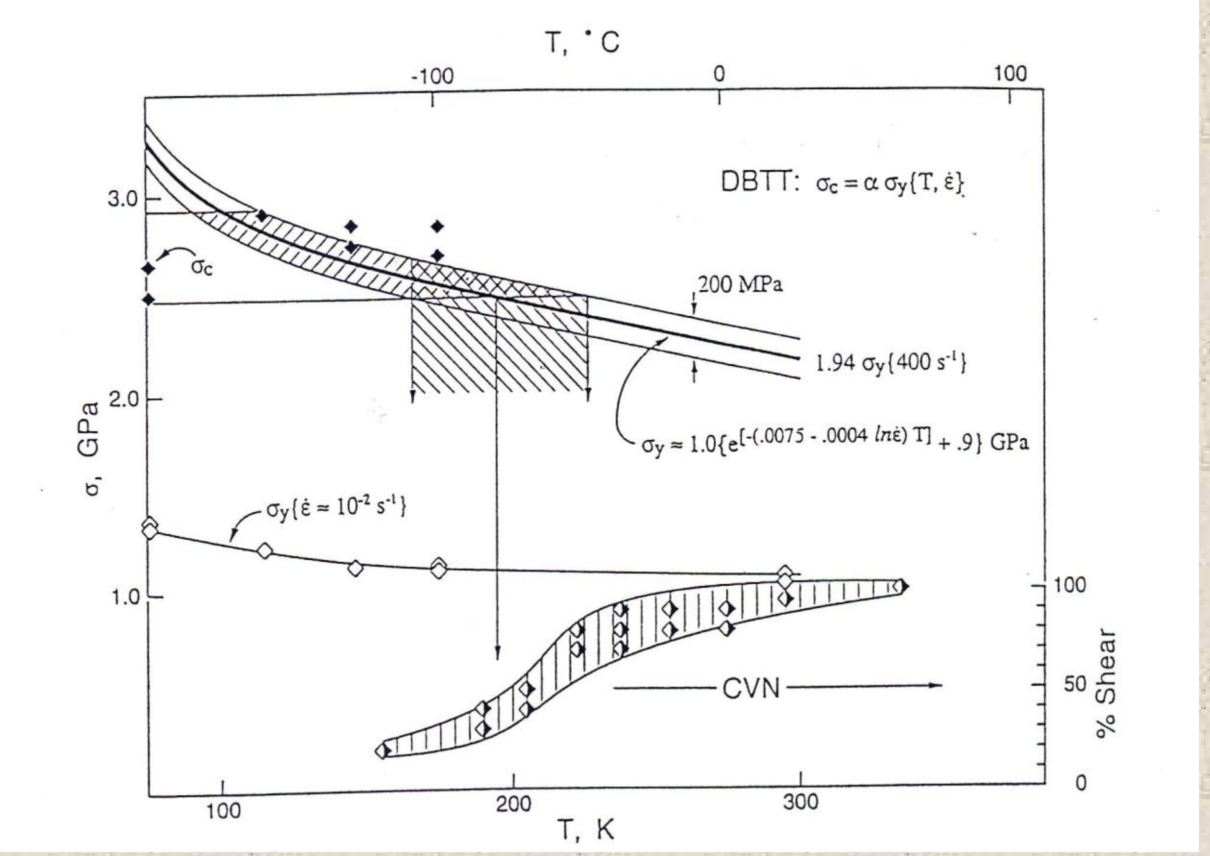
Impact

THE WASHINGTON POST

SUNDAY, FEBRUARY 16, 1997 A27

"Impact is the most fundamental of all geological processes," said Eugene Shoemaker, of the U.S. Geological Survey, who co-discovered the comet that hit Jupiter. "It's how the Earth was formed."

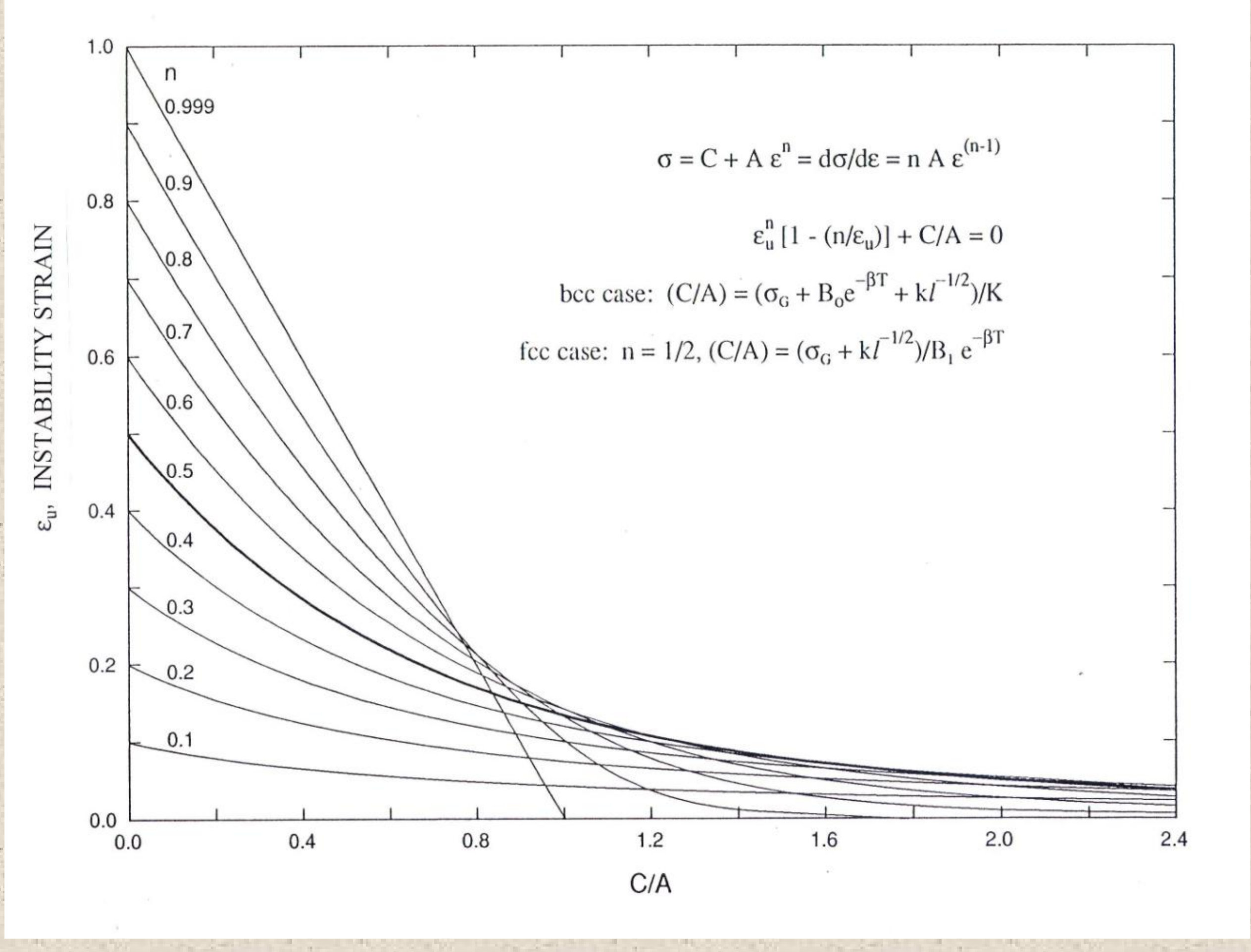
Charpy impact transition temperature



$\alpha'\sigma_y = \sigma_C$ at $T_C = (1/\beta)[\mathbf{ln}\alpha' B - \mathbf{ln}\{(k_C - \alpha' k_y) + (\sigma_{0C} - \alpha'\sigma_{0G})\ell^{1/2}\} - \mathbf{ln}\ell^{-1/2}]$

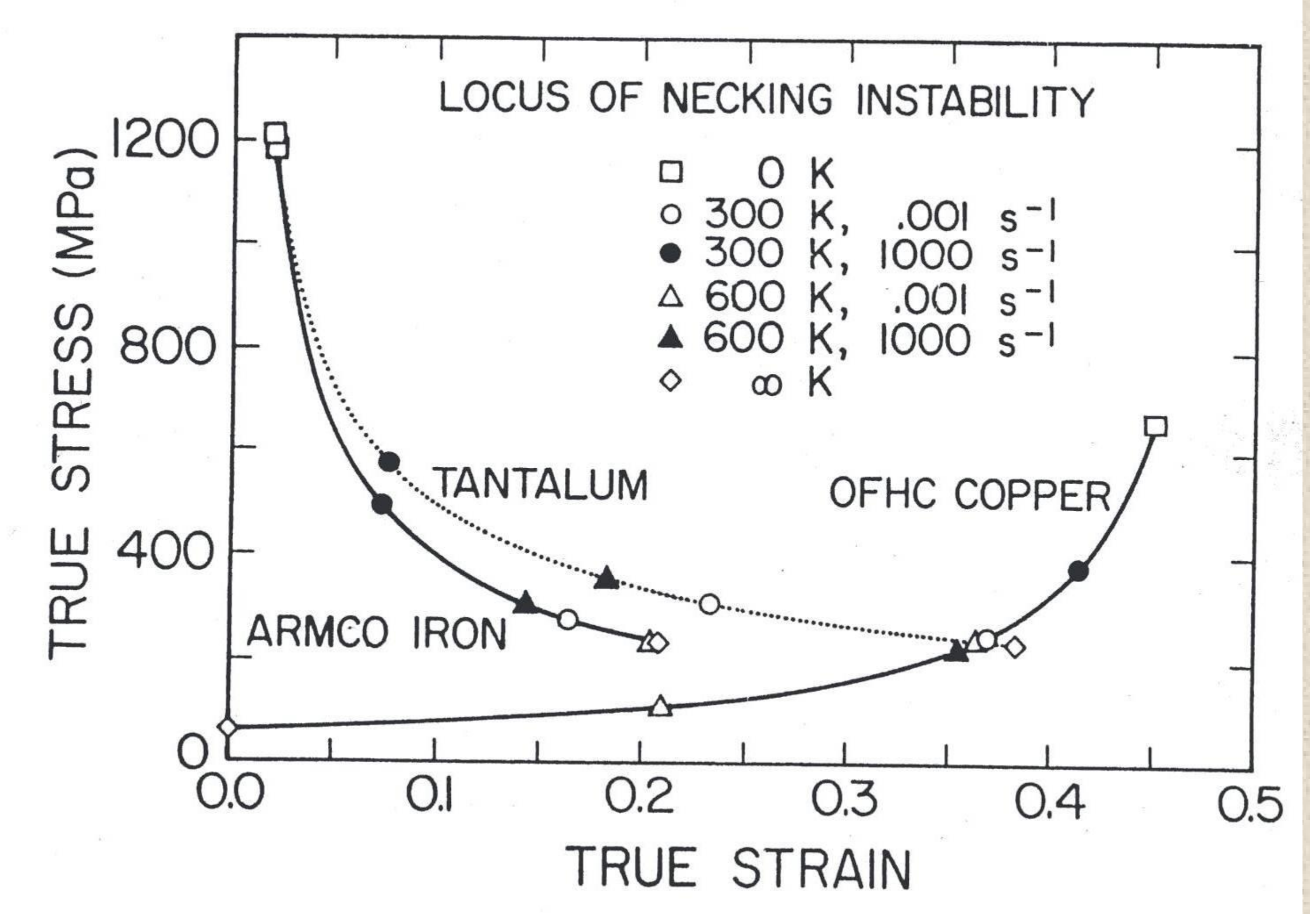
R.W. Armstrong, *Eng. Fract. Mech.*, **28**, 529-538 (1987)

Tensile plastic instability: $\sigma = d\sigma/d\varepsilon$



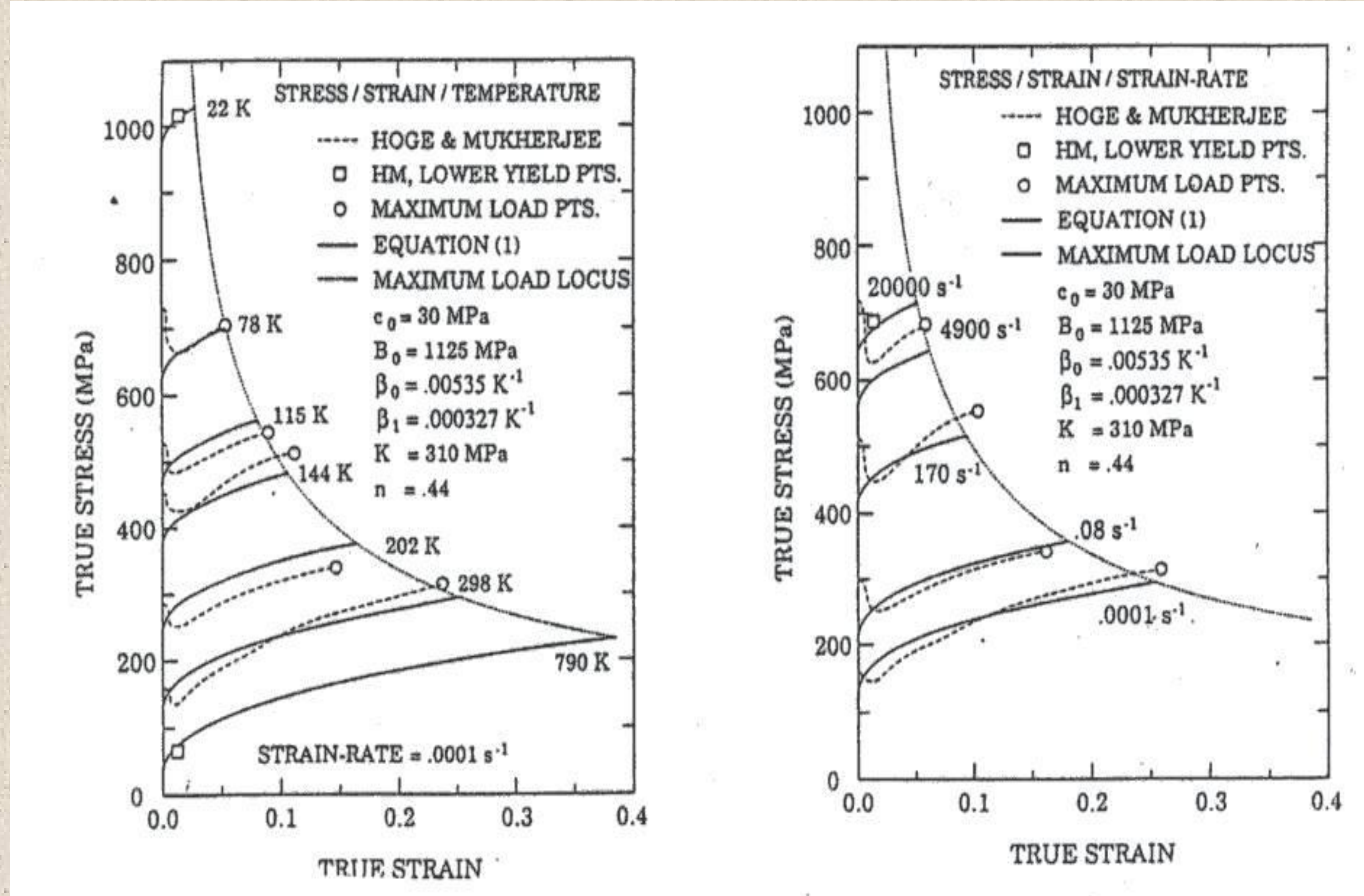
R.W. Armstrong and F.J. Zerilli, *Mech. Mater.*, **17**, 319-327 (1994)

BCC vs. FCC plastic instability dependences



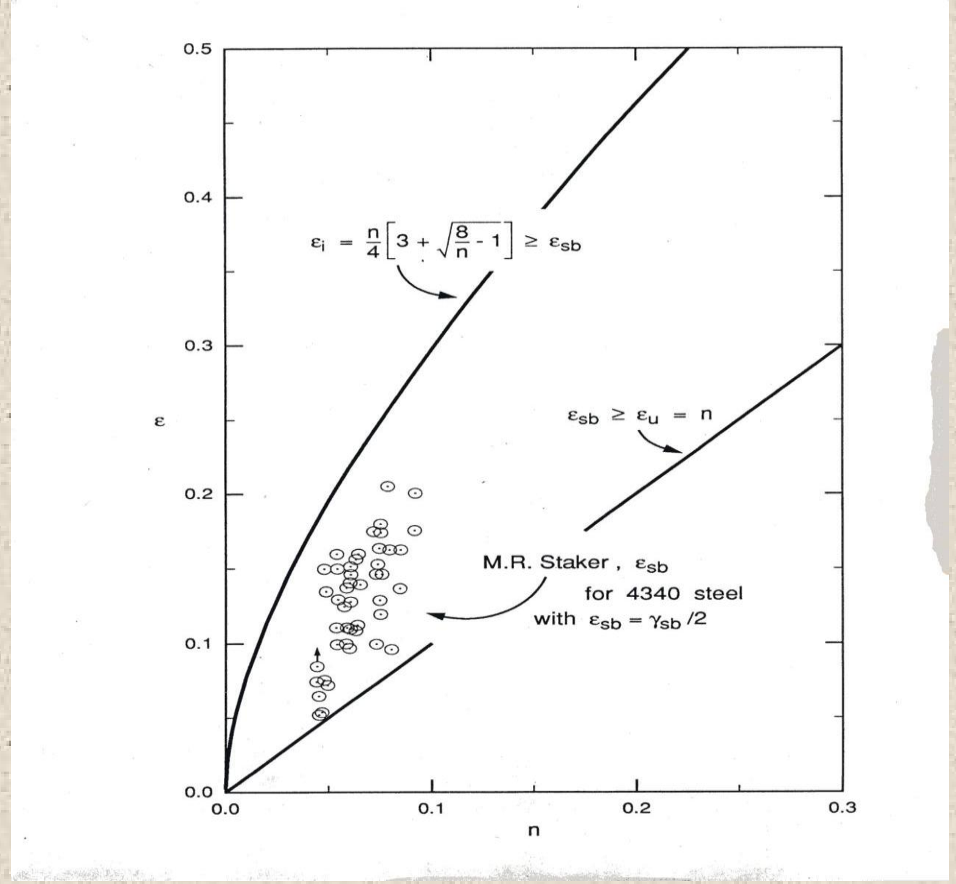
R.W. Armstrong and F.J. Zerilli, *Mech. Mater.*, **17**, 319-327 (1994)

Tantalum: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$



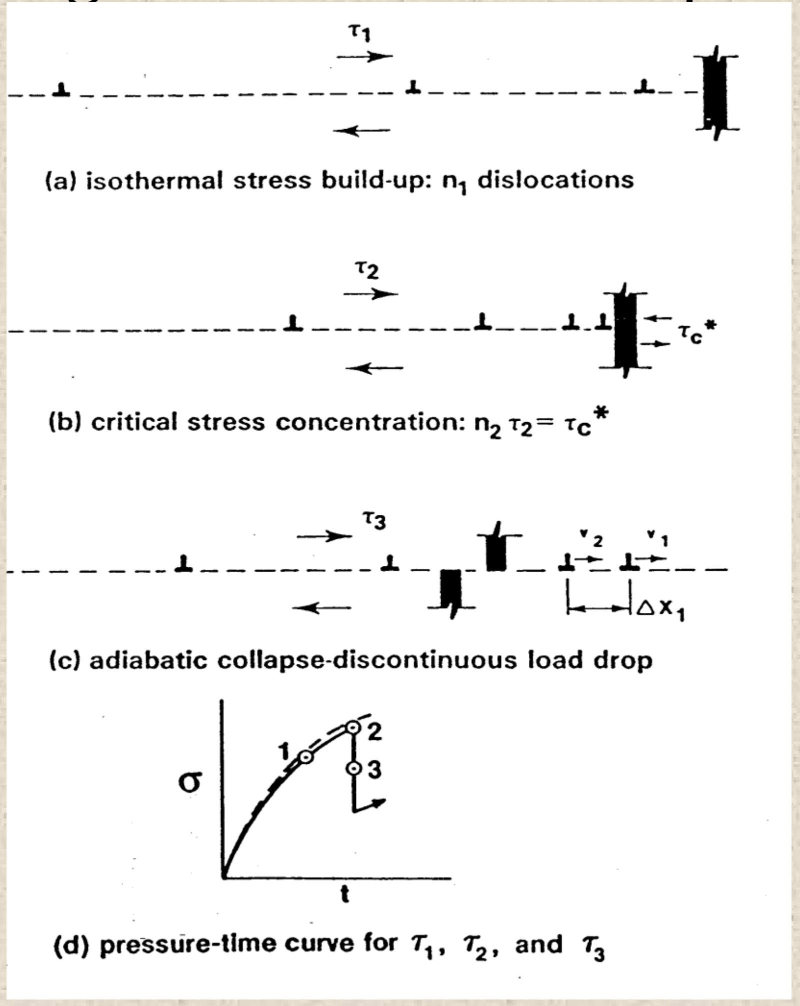
F.J. Zerilli and R.W. Armstrong. *J. Appl. Phys.*, **68**, 1580-1591 (1990)

Shear banding based on $\sigma = K\varepsilon^n$ and $\sigma = d\sigma/d\varepsilon$; from $(dP/d\ell_0) = 0$ and raised to $(d^2P/d\ell_0^2) = 0$



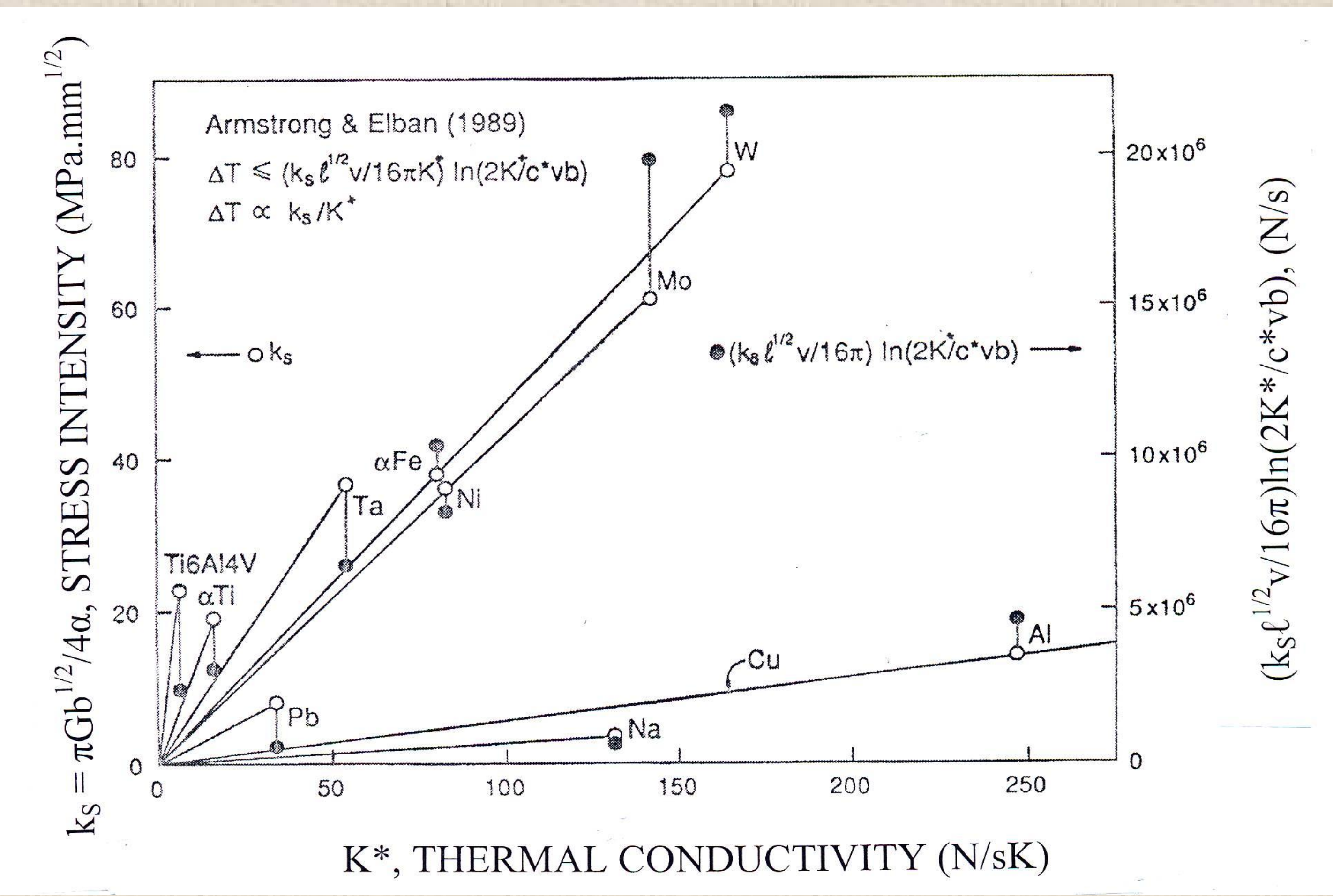
Measurements from M.R. Staker, *Acta Metall.*, **29**, [4], 683-689 (1981)

Shear banding from a dislocation pile-up avalanche



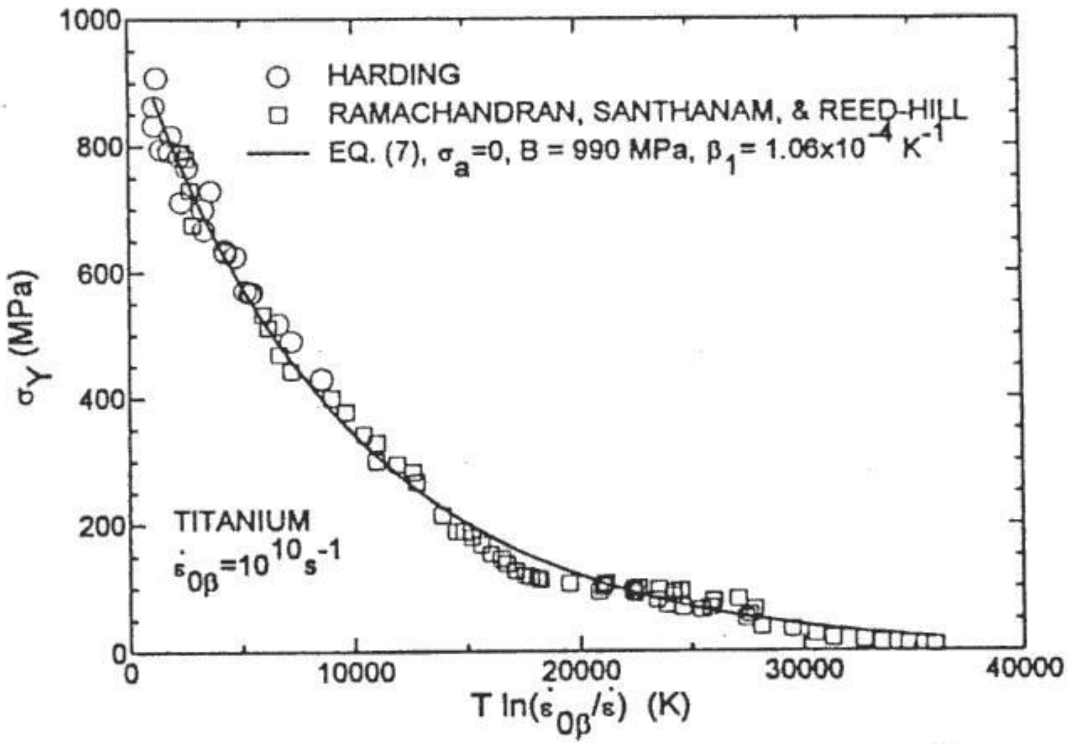
R.W. Armstrong, C.S. Coffey, and W.L. Elban, *Acta Metall.*, **30**, 2111-2118 (1982)

Shear band susceptibility from the pile-up avalanche model



R.W. Armstrong and F.J. Zerilli, *Mech. Mater.*, **17**, 319-327 (1994)

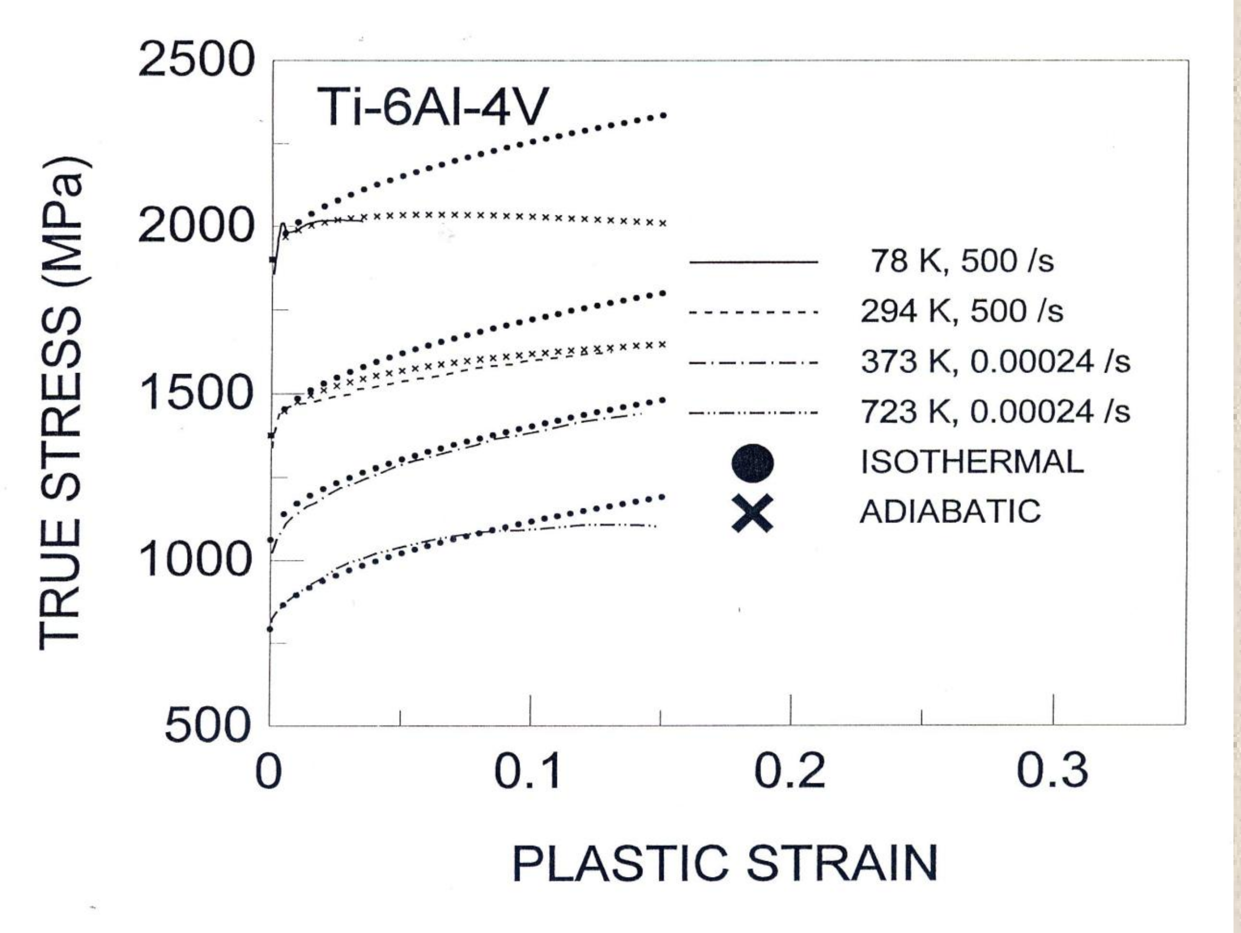
Titanium: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$



$$\sigma_Y = \sigma_a + Be^{-\beta_1 \xi_\beta} \quad \text{with} \quad \xi_\beta = T\ln\left(\frac{\dot{\epsilon}_{0\beta}}{\dot{\epsilon}}\right)$$

F.J. Zerilli and R.W. Armstrong, *Shock Compression of Condensed Matter – 1995*, edited by S.C. Schmidt and W.C. Tao (Amer. Inst. Phys., N.Y., 1996) **CP 370**, pp. 315-318

Isothermal and adiabatic stress strain curves for Ti6Al4V material



F.J. Zerilli and R.W. Armstrong, *Shock Compression of Condensed Matter – 1995*, edited by S.C. Schmidt and W.C. Tao (Amer. Inst. Phys., N.Y., 1996) pp. 315-318

Copper: $\sigma = \sigma\{(d\varepsilon/dt), T, \ell^{-1/2}\}$

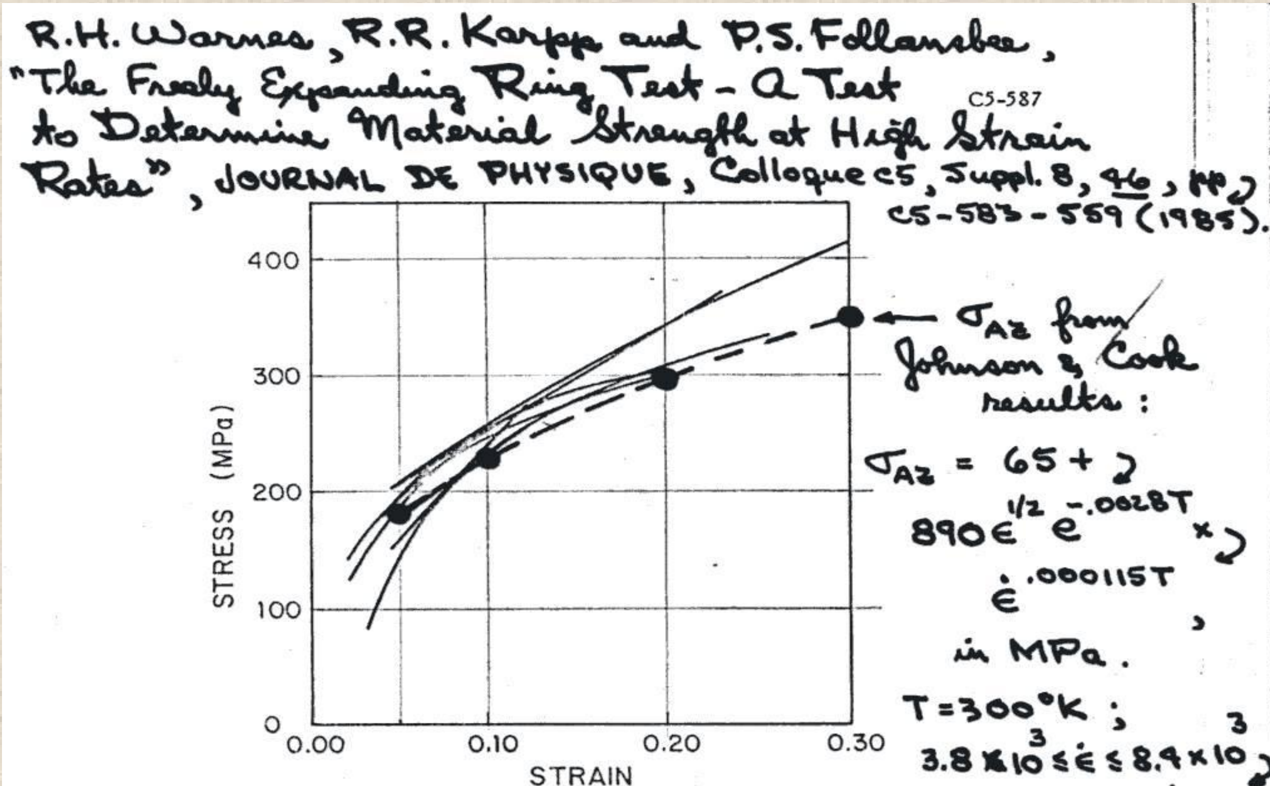
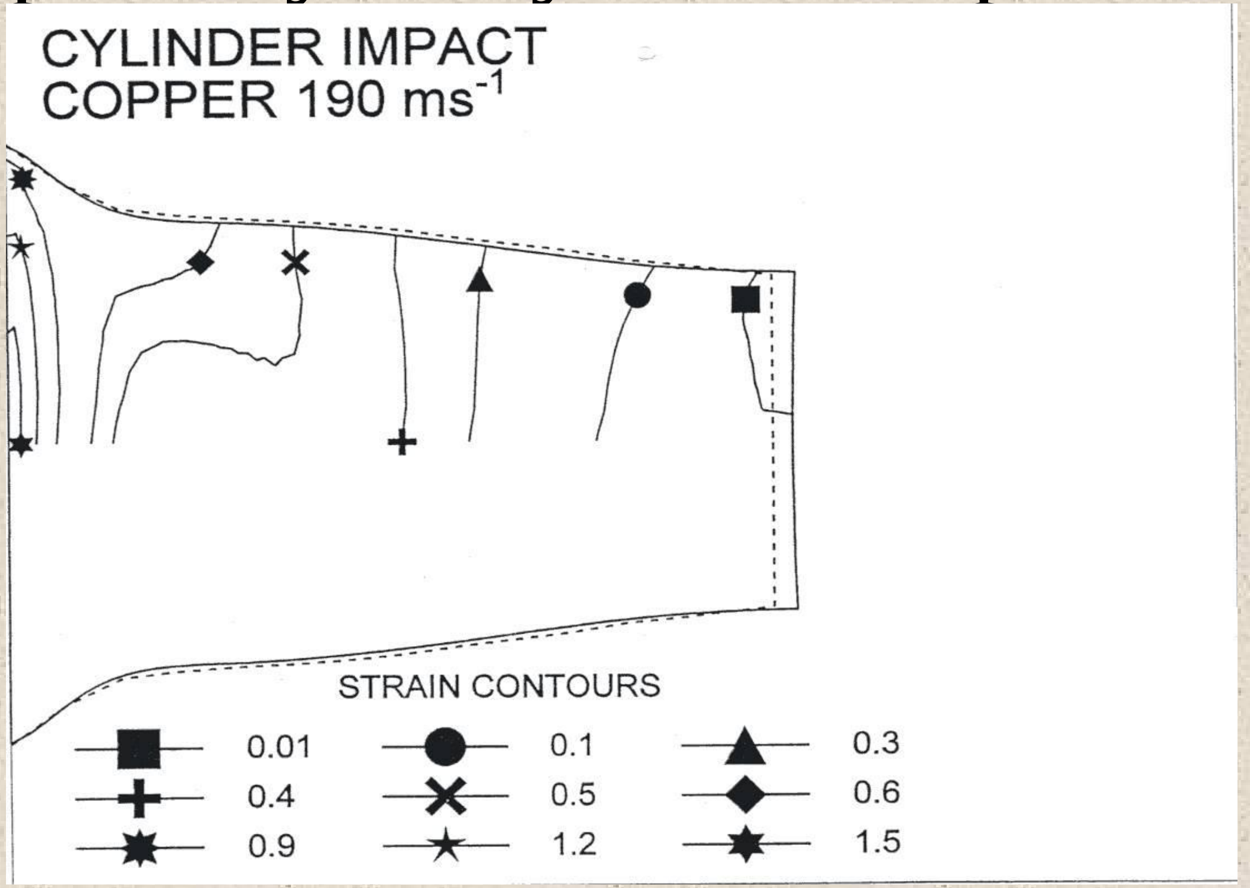


Fig. 4 - The stress-strain behavior of copper determined from the data shown in Fig. 3 and analyzed using Eq. (1) and either Eq. (4) or Eq. (5).

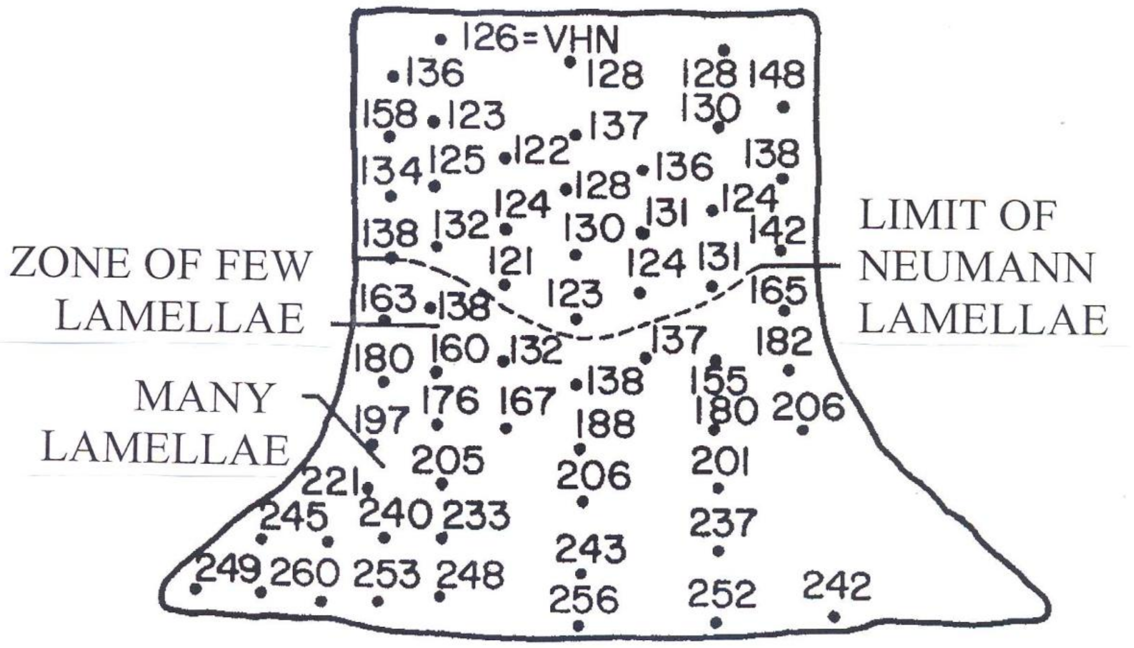
F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.*, **61**, 1816-1825 (1987); see P.S. Follansbee and U.F. Kocks, Acta Metall., **36**, 81-93 (1988).

Copper Taylor cylinder impact result



F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.*, **61**, 1816-1825 (1987)

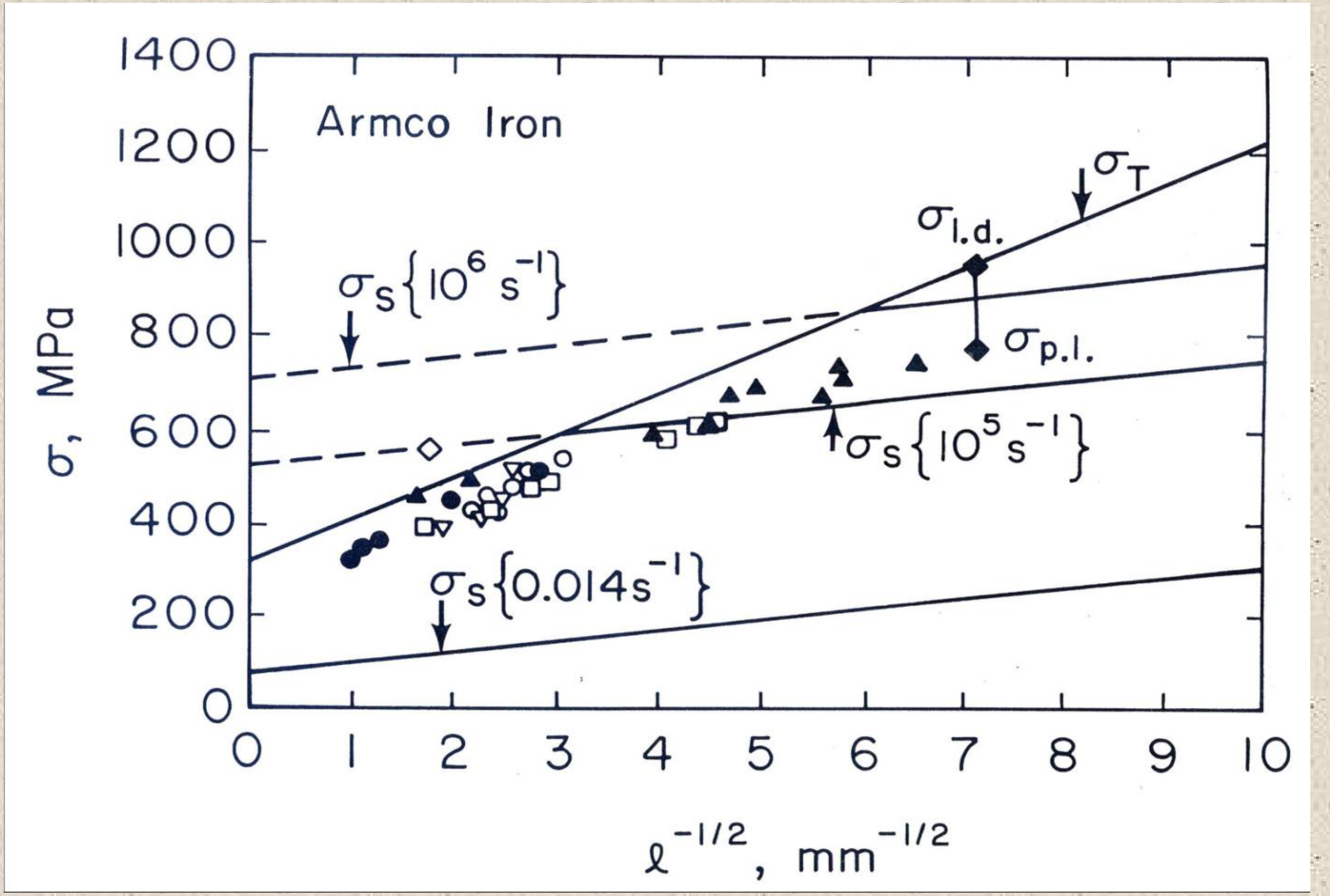
Original Taylor cylinder test result on mild steel



MILD STEEL CYLINDER. IMPACT VELOCITY 338 m/s

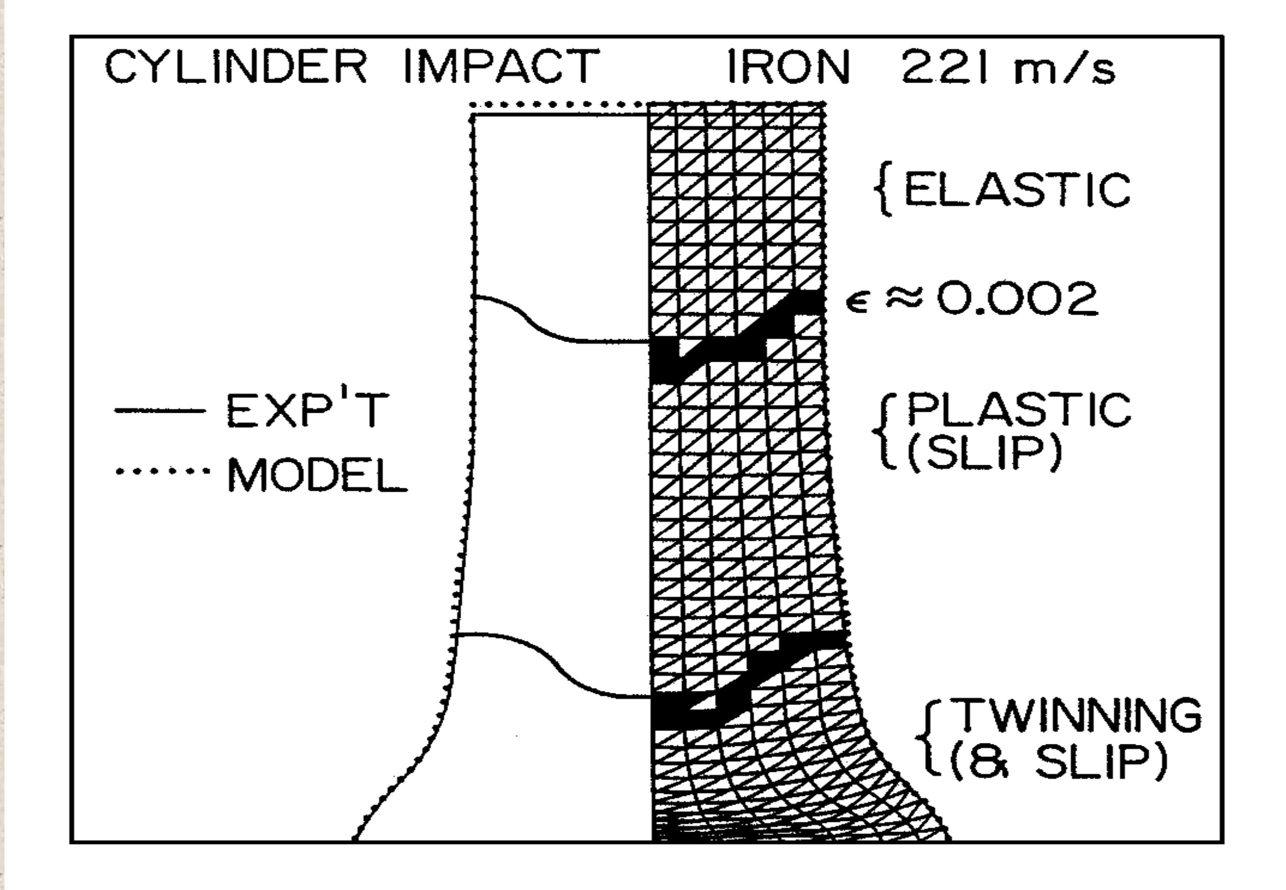
W.E. Carrington and M.L.V. Gaylor, *Proc. Roy. Soc. London A*, **194A**, 323-331 (1948)

SHPB twinning measurements compared to Z-A slip calculations



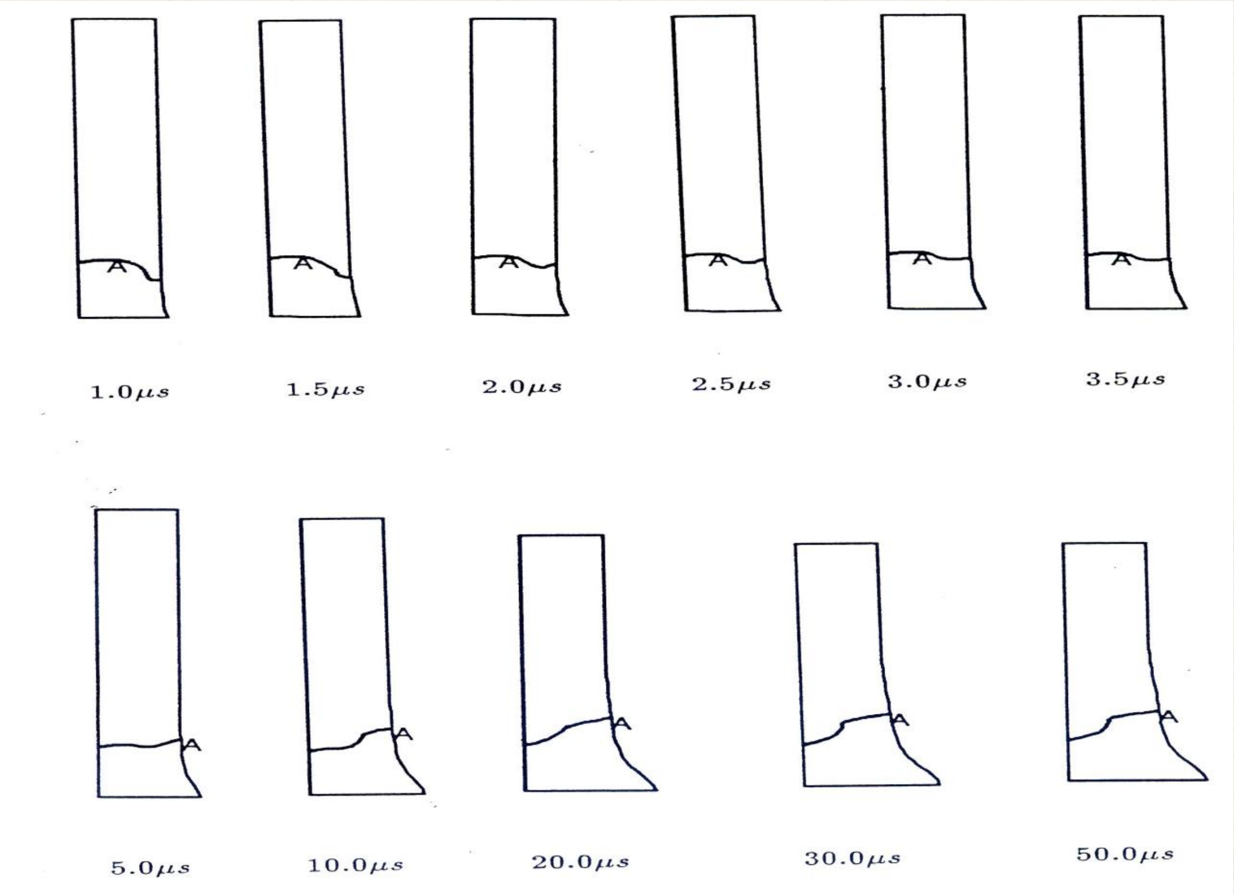
R.W. Armstrong and F.J. Zerilli, *J. Phys. Fr. Colloq.*, **49**, [C3], 529-534 (1988)

Armco iron Taylor test involving twinning and slip



F.J. Zerilli and R.W. Armstrong, *Shock Compression of Condensed Matter*, edited by S.C. Schmidt and N.C. Holmes (Elsevier Sci. Publ. B.V., N.Y., 1988) pp. 273-277.

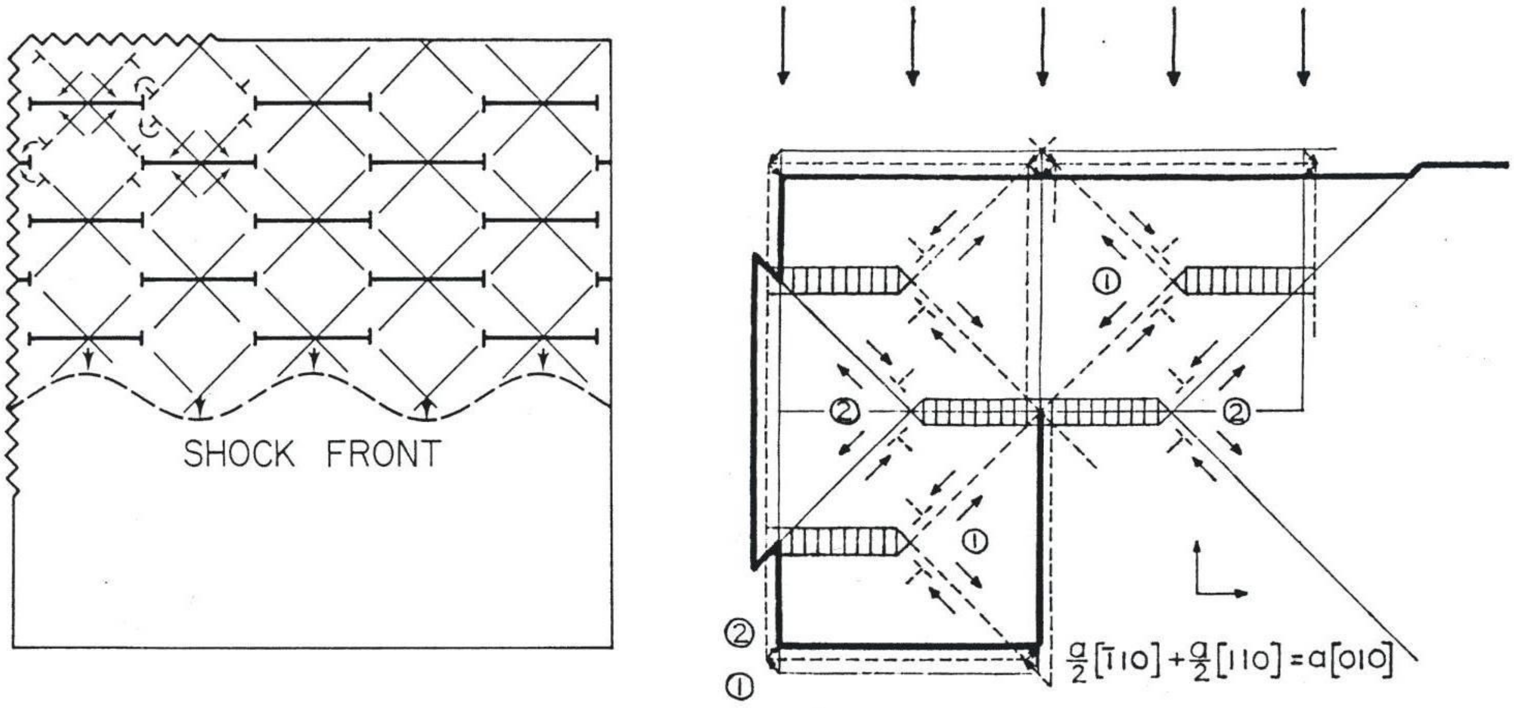
Sequential twinning and slip in the Taylor cylinder impact test



J. B. McKirgan, *"Microstructurally-based EPIC simulations of Taylor impact tests"*, M.Sc. Thesis, University of Maryland, 1990

Model for a propagating shock front

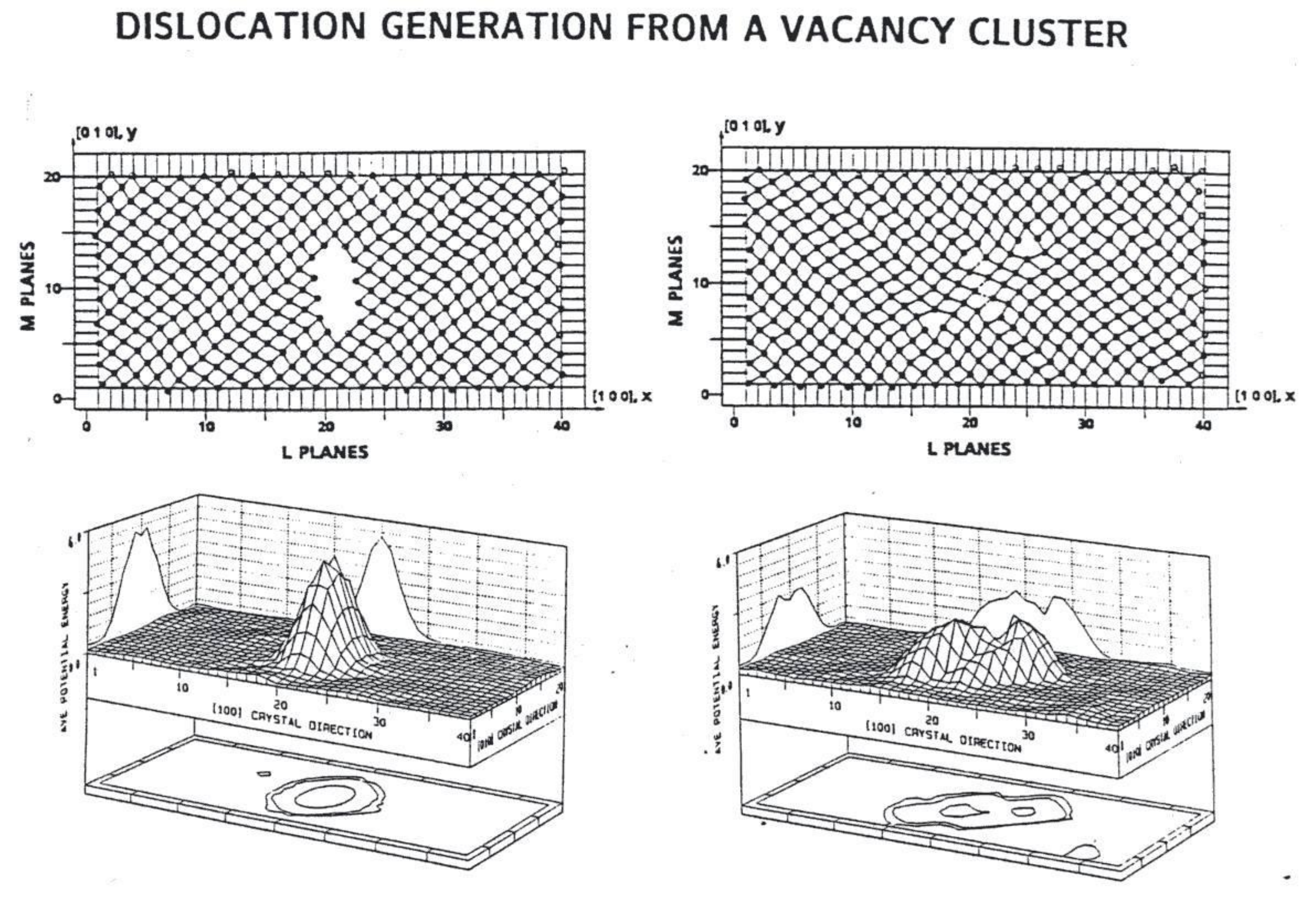
$\sigma = P\{(1 - \nu)/(2 - \nu)\}$ for one-dimensional strain



REACTED DIPOLES FROM SHEAR AT THE SHOCK FRONT

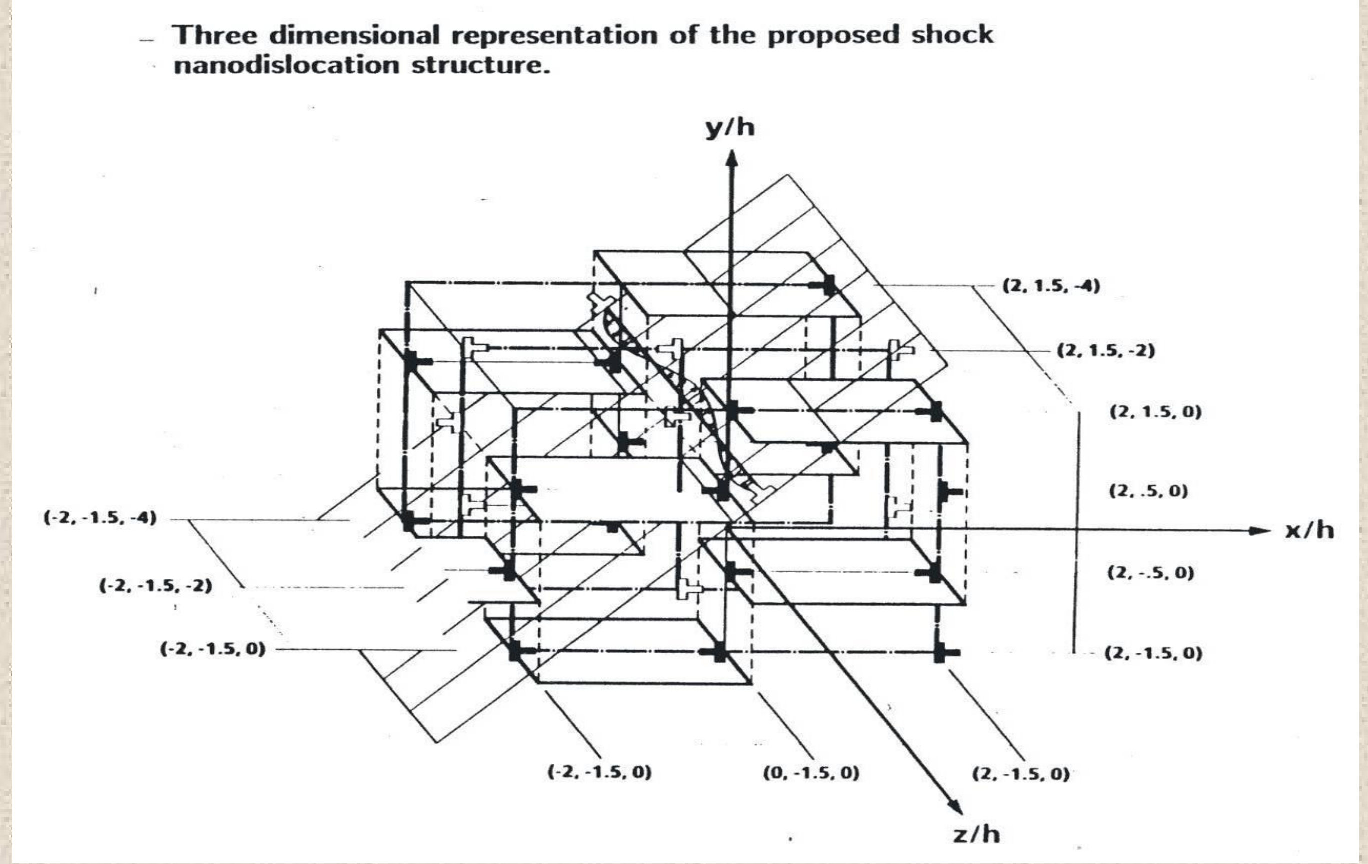
F.A. Bandak, D.H. Tsai, R.W. Armstrong and A.S. Douglas, *Phys. Rev. B*, **47**, 11681-11687 (1993)

MD model of shock-induced dislocation dipole structures



F.A. Bandak, R.W. Armstrong, and A.S. Douglas, *Phys. Rev. B*, **46**, 3228-3235 (1992).

Model of 3-D post-shock deformation



F.A. Bandak, R.W. Armstrong, and A.S. Douglas, *Phys. Rev. B*, **46**, 3228-3235 (1992)

Analogy with "channeling" of deformation in post-deformation of neutron-irradiated materials

Transition from strain rate control by dislocation velocity to control by dislocation generation

For dislocation velocity control:

$$(d\varepsilon/dt) = (1/m)\rho b\nu$$

$$\sigma = \sigma_G + B\mathbf{exp}[-\beta T] + B_0[\varepsilon r(1 - \mathbf{exp}\{-\varepsilon/\varepsilon_r\})]^{1/2}\mathbf{exp}[-\alpha T] + k_\varepsilon \ell^{-1/2}$$

in which

$$(\beta,\alpha) = (\beta_0,\alpha_0) - (\beta_1,\alpha_1)\mathbf{ln}(d\varepsilon/dt)$$

For dislocation generation control:

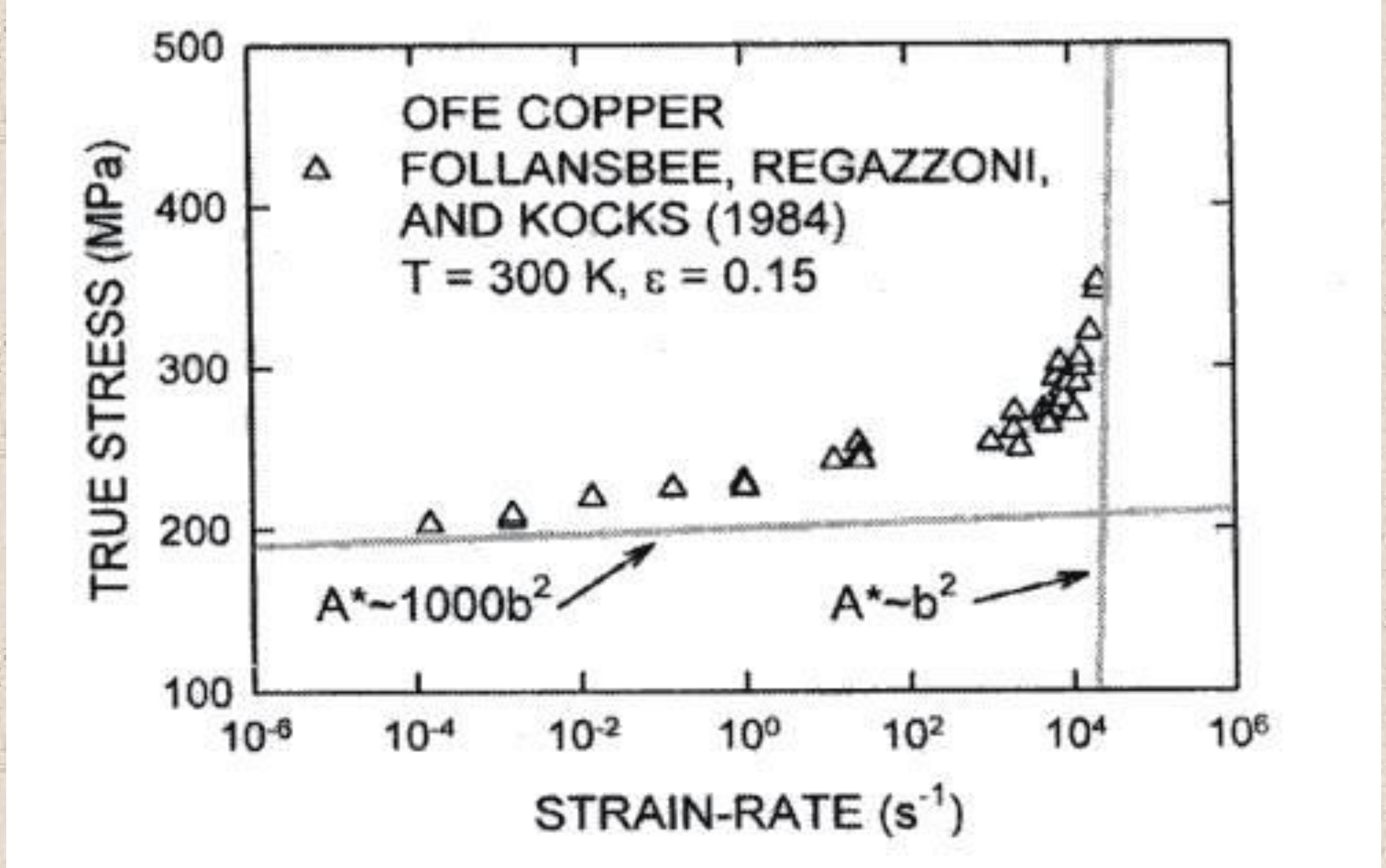
$$(d\varepsilon/dt) = (1/m)b(d\rho/dt)\Delta x_d$$

for which, at limiting small value of $v^* \sim b^3$, and $m = 2$

$$\sigma_{Th} = (2G_{0G}/v^*) - (2k_BT/v^*)\,[\mathbf{ln}\{(d\varepsilon/dt)_0/(d\varepsilon/dt)\}]$$

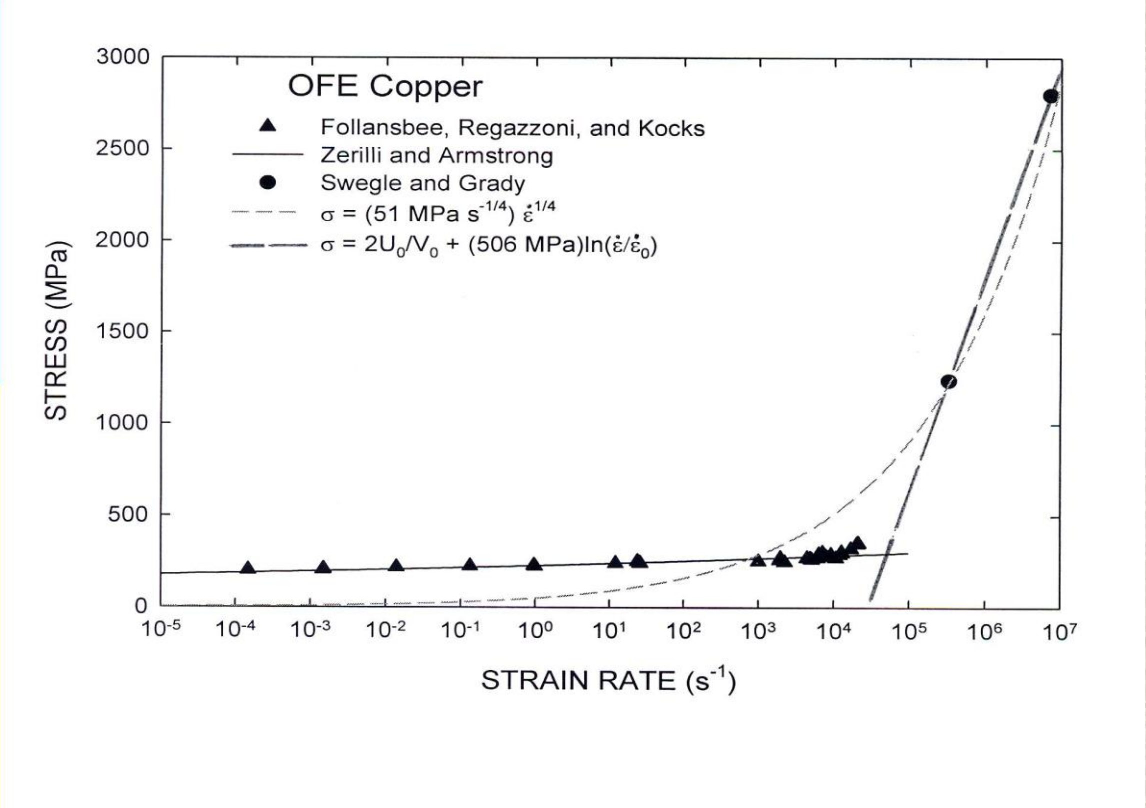
R.W. Armstrong, W. Arnold, and F.J. Zerilli, *Metall. Mater. Trans. A*, **38A**, 2605-2610 (2007)

SHPB indication of dislocation generation



R.W. Armstrong, W. Arnold and F.J. Zerilli, *Metall. Mater. Trans. A*, **38A**, 2605-2610 (2007)

Connection of Follansbee *et al* results with Swegle and Grady shock measurements



R.W. Armstrong, W. Arnold, and F.J. Zerilli, *Metall. Mater. Trans. A*, **38A**, 2605-2610 (2007).

Grain Size Dependent Slip/Twinning Transition in Iron at the Hugoniot Elastic Limit (HEL)

Hall-Petch Relations:

Slip

$$\sigma = B\exp(-\beta T) + A\varepsilon^n + \sigma_G + k_y \ell^{-1/2}$$

$$\beta = \beta_0 - \beta_1 \ln(d\varepsilon/dt)$$

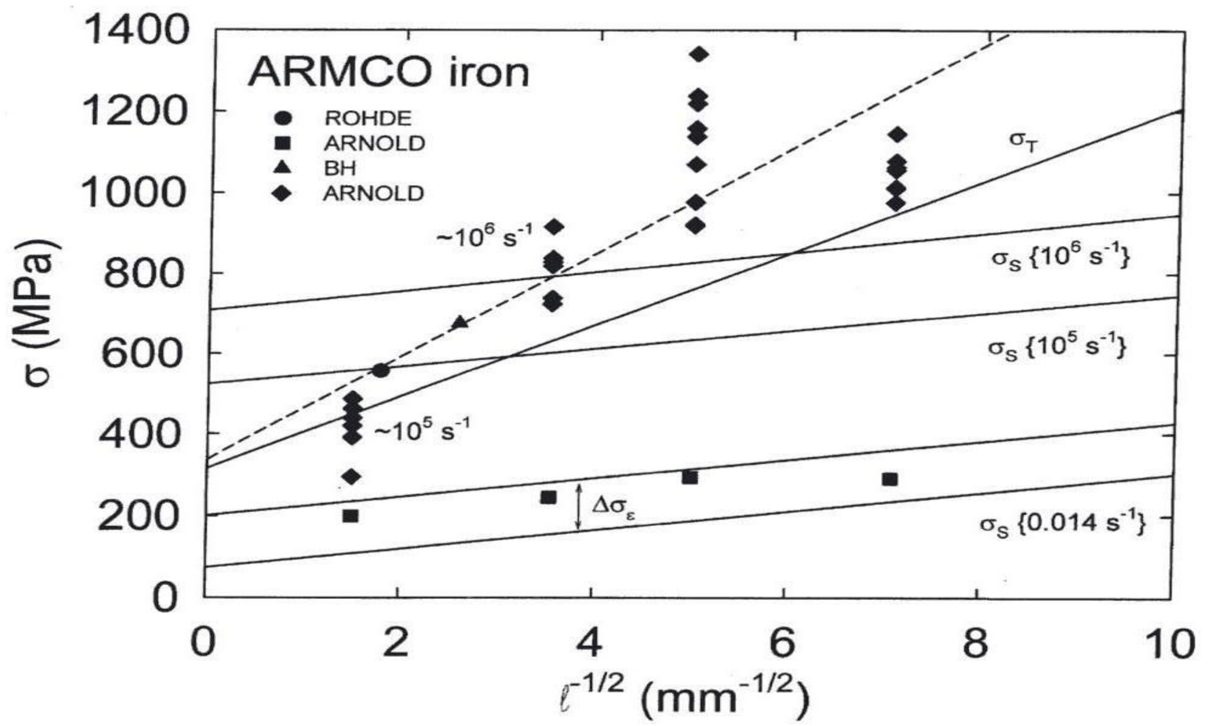
Deformation Twinning

$$\sigma_T = \sigma_{T0} + k_T \ell^{-1/2}$$

References

1. F.J. Zerilli, Metall. Mater. Trans. A **35A**, 2547 (2004).
2. R.W. Armstrong and F.J. Zerilli, J. Phys. Coll. **49**, (C3), 529 (1988).

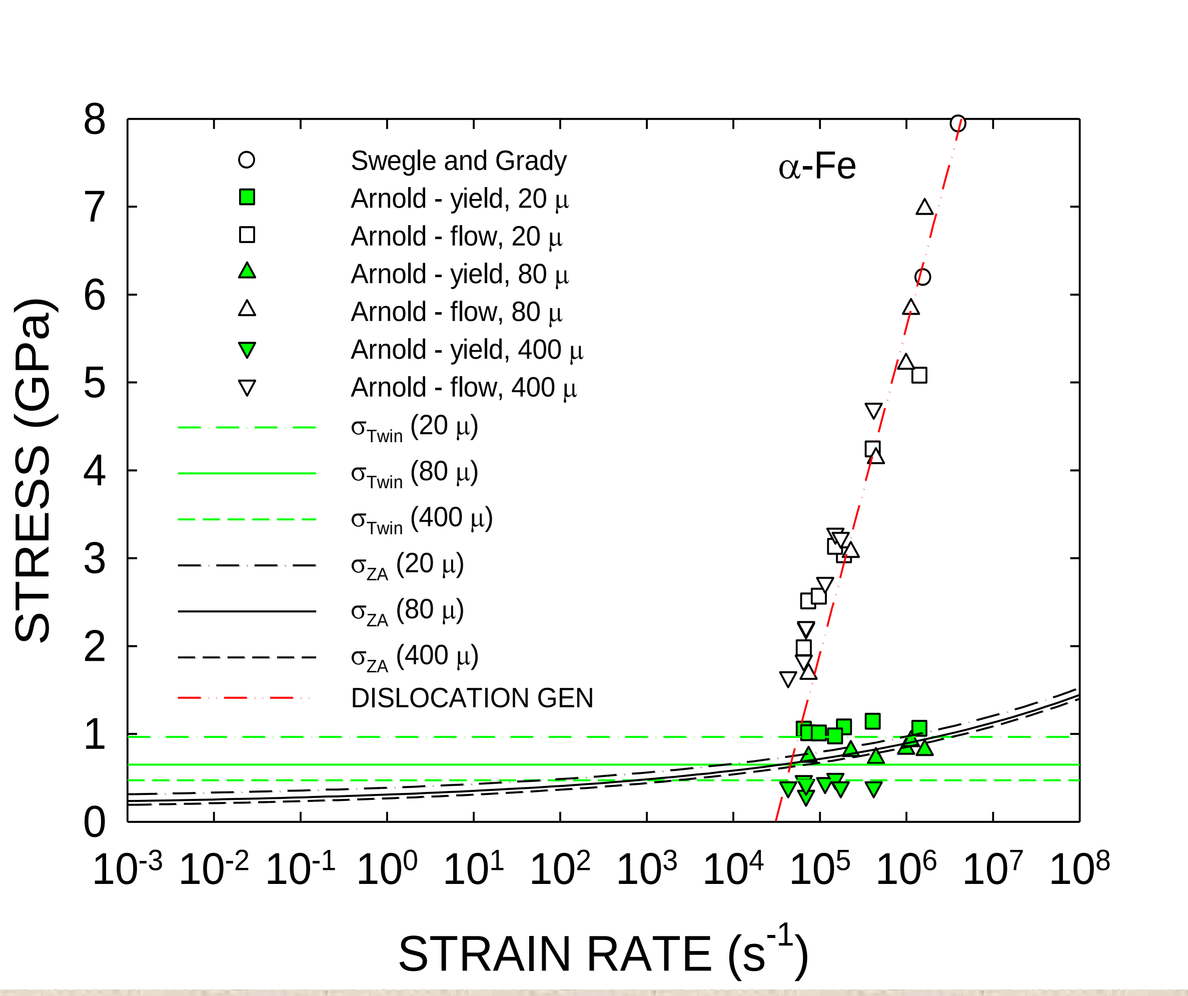
Pre-shock Hardness and HEL Measurements as Compared with Model Slip and Twinning Equations



R.W. Armstrong, W. Arnold, and F. J. Zerilli, *Metall. Mater. Trans. A*, **38A**, 2605-2610 (2007);

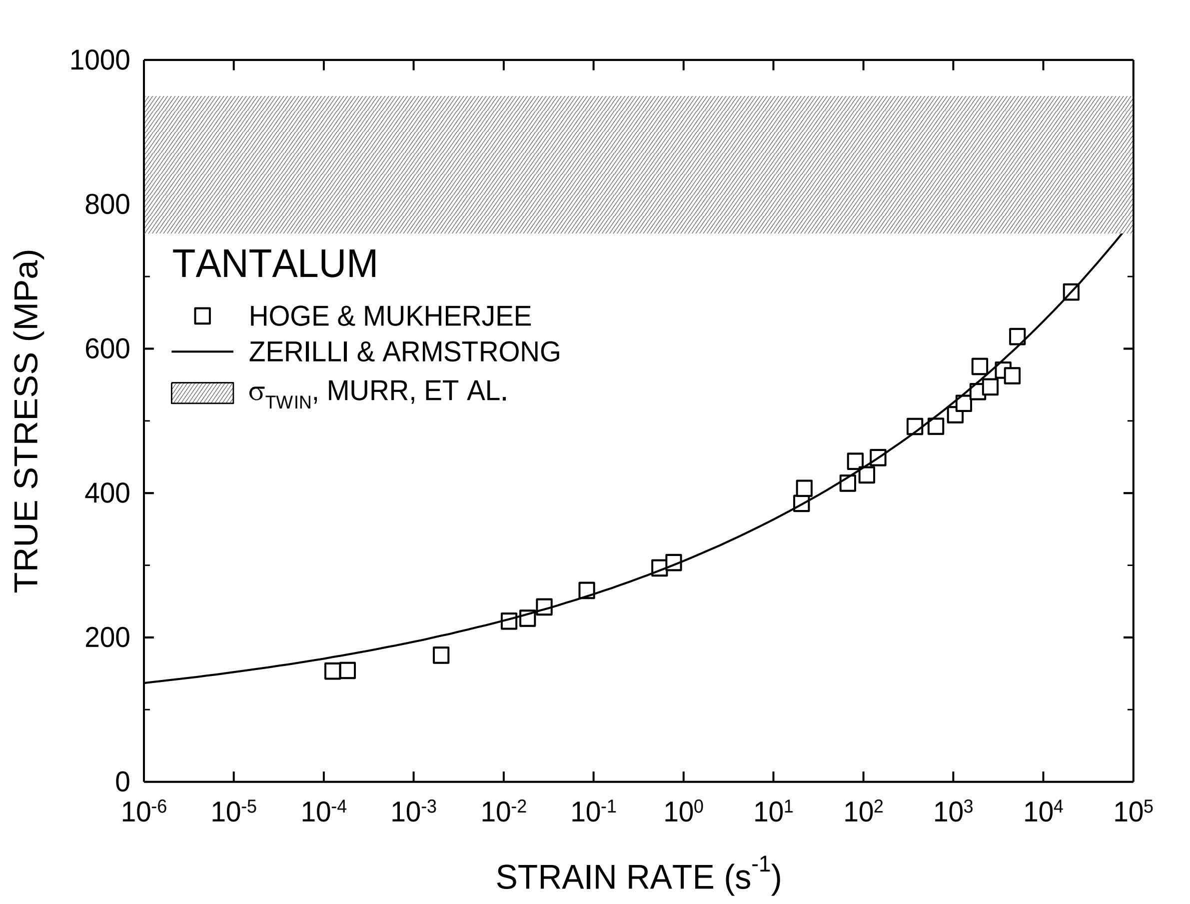
W. Arnold, *Dynamisches Werkstoffverhalten von Armco-eisen bei Stosswellenbelastung*, Fortschrittberichte VDI, 5, 247,VDI-Verlag GmbH, Duesseldorf, DE, 1992

Shocked Armco iron at different grain sizes and plastic strain rates



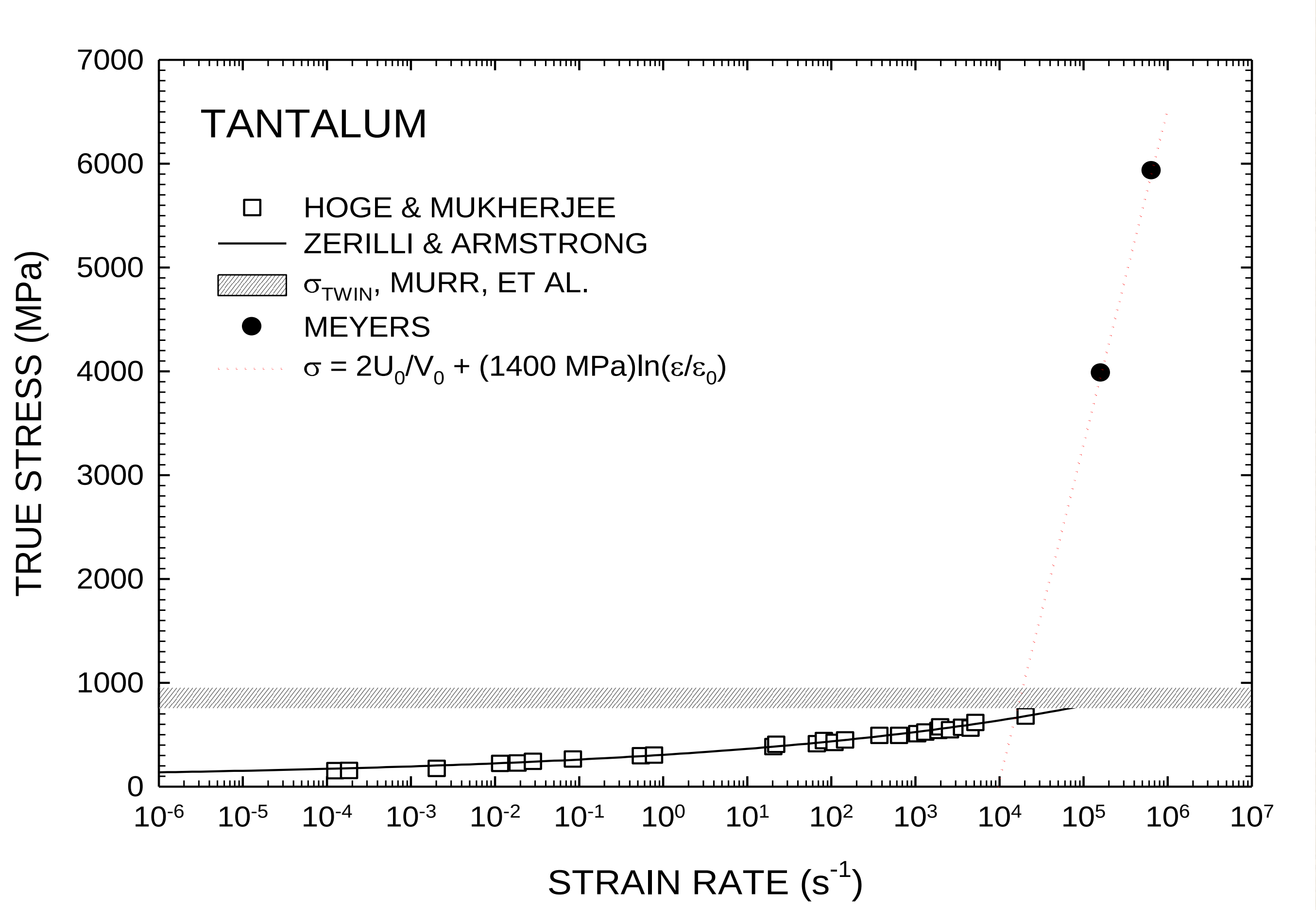
R.W. Armstrong, W. Arnold and F.J. Zerilli, *J. appl. Phys.*, **105**, 023511 (2009)

Tantalum



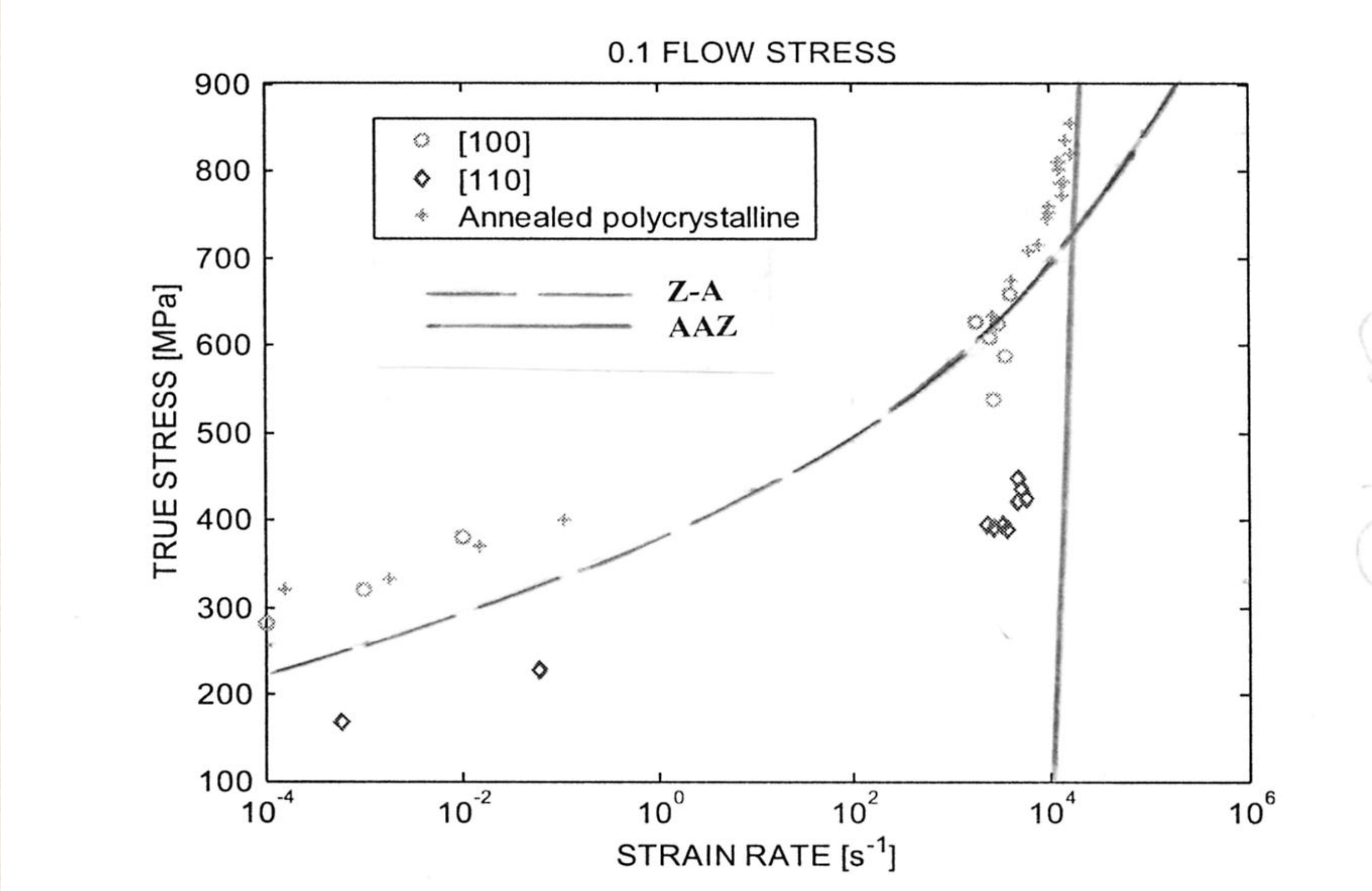
F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.*, **68**, 1580-1591 (1990)

Tantalum Conventional and Shock Results



R.W. Armstrong, W. Arnold and F.J. Zerilli, *Metall. Mater. Trans. A*, **38A**, 2605-2610 (2007)

Ta single crystal/polycrystal SHPB results



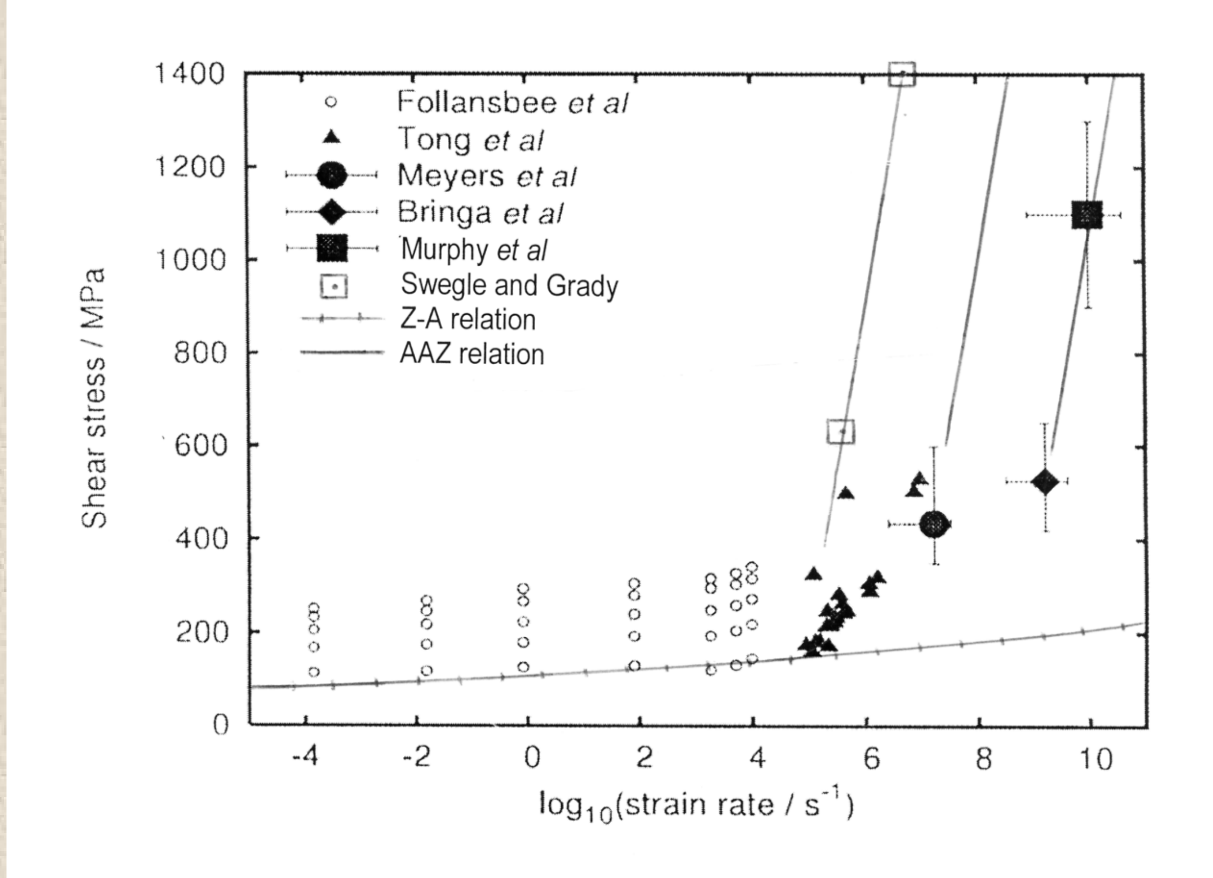
D. Rittel, M.L. Silva, B. Poon and G. Ravichandran, *Mech. Mater.*, **41**, 1323-1329 (2009); R.W. Armstrong and F.J. Zerilli, *J. Phys. D: Appl. Phys.*, **43**, 492002 (2010)

Critical activation volumes, v*, under shock loading

Metal	b (10^{-10} m)	$2kT/b^3$ (MPa)	$\Delta\sigma/\Delta\ln[d\varepsilon/dt]$ (MPa)	V* (b^3)**
Cu	2.55	500	505	1.0
Al	2.86	354	252	1.4
α-Fe	2.48	542	1400	0.4
Ta	2.84	361	1200	0.3

** $V^* = mkT(\partial \ln[d\varepsilon/dt]/\partial\sigma)_T$

Laser-shocked deformation of copper



W.J. Murphy *et al.*, *J. Phys.: Condens. Matter*, **22**, 065404 (2010); R.W. Armstrong and F.J. Zerilli. *J. Phys. D: Appl. Phys.*, **43**, 492002 (2010)

The strain and strain rate from combined dislocation displacements and generations

The strain from displaced and generated dislocations

$$\Delta\varepsilon = (1/m)[\rho_N b\Delta x_N + \Delta\rho_G b\Delta x_d + \rho_G b\Delta x_d]$$

The strain rate is obtained then for the presumed time-dependent parameters as

$$(d\varepsilon/dt) = (1/m)[\rho_N b\nu_N + (d\rho_G/dt)b\Delta x_d + \rho_G b\nu_G]$$

with neglect of $(d\rho_N/dt)$. If it were assumed that $\nu_N = \nu_G = \nu^*$, and ρ_N and ρ_G could be combined as ρ_T then

$$(d\varepsilon/dt) = (1/m)[(d\rho_G/dt)b\Delta x_d + \rho_T b\nu^*]$$

Shockless isentropic compression experiments (ICEs)

The resident dislocation density is required to “carry the load”, and because ρ_N is low, ν_N is so high as to be controlled by “drag”!

$$\sigma_{Th} = \{1 – [c(d\varepsilon/dt)/\beta_1\sigma_{Th}]^{-\beta 1T} \}[B\mathbf{exp}(-\beta T)]$$

in which

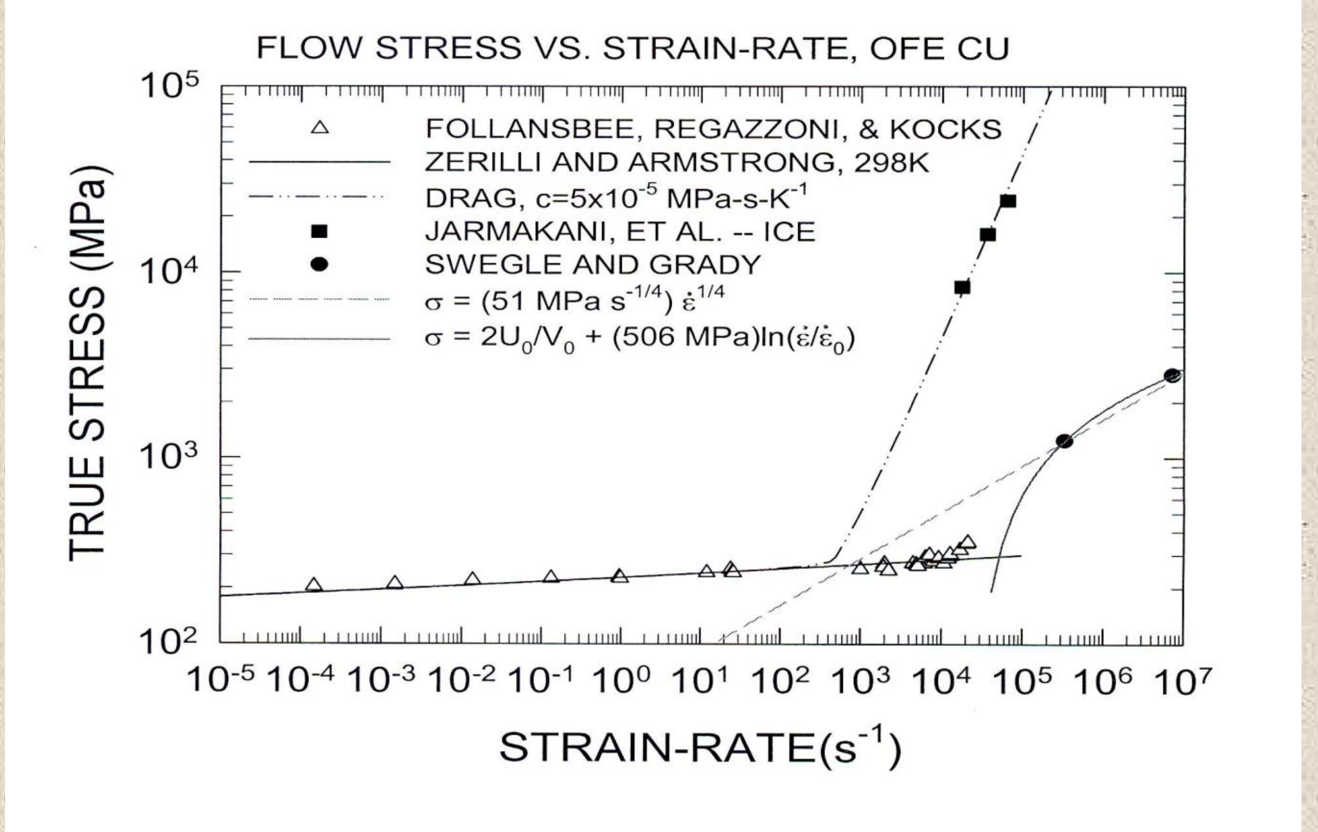
$$c = c_0 m^2\beta_1/\rho b^2 \quad \text{and} \quad b\tau_{TH} = c_0\nu.$$

At high $(d\varepsilon/dt)$:

$$\sigma_{Th} = (c_0 m^2/\rho b^2)(d\varepsilon/dt)$$

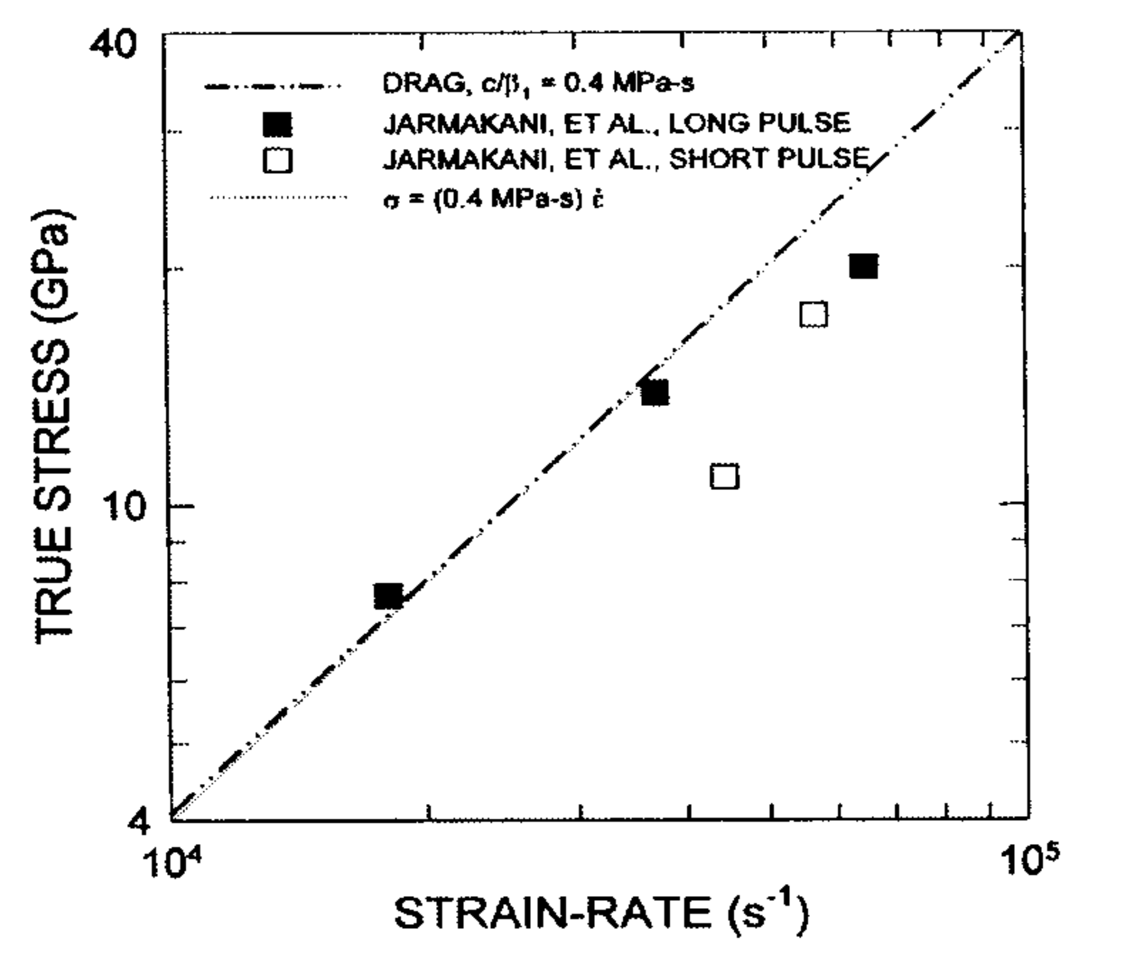
F.J. Zerilli and R.W. Armstrong, *Acta Mater.*, **40**, 1803-1808 (1992);
R.W. Armstrong, W. Arnold and F.J. Zerilli, *J. Appl. Phys.* 105, 023511 (2009)

Copper SHPB, Shock, and ICE results



H. Jarmakani et al., *Mater. Sci. Eng. A*, **463**, 249 (2007);
R.W. Armstrong, W. Arnold and F.J. Zerilli, *J. Appl. Phys.*, **105**, 023511 (2009)

Drag-controlled shockless ICE results for copper



R.W. Armstrong, W. Arnold and F.J. Zerilli, *J. Appl..Phys.*, **105**, 023511 (2009)

Strength of Material Dynamics
SUMMARY

1. An introduction has been given on a dislocation mechanics basis to coupling of model thermal activation – strain rate analysis (TASRA) and grain size related, dislocation pile-up, constitutive relations.
2. Attention was directed to the strain rate, temperature, and grain size dependencies that are established for evaluations of such material strength properties as: (i) conventional stress – strain; (ii) ductile-brittle transition; (iii) Charpy v-notch impact; (iv) plastic instability; (v) shear banding; (vi) Taylor cylinder impact; (vii) shock; and, (viii) shockless isentropic compression experiments (ICEs).
3. Beyond giving emphasis to the dislocation velocity and generation being thermally activated processes at the crystal lattice scale, the fuller description involves key features of (i) dislocation pile-up associated internal stress concentrations, (ii) relief needed for dislocation generations at all lattice points along a propagating shock front; and, (iii) trade-offs between dislocation density and velocity in distinguishing between very different strength properties at comparable shock and shockless deformation rates.