60 Years of Hall-Petch: Past to Present Nano-Scale Connections

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The inverse square root of grain size dependence reported in pioneering researches by Hall and by Petch for the yield and cleavage fracture stresses of iron and steel materials has spread to elucidation of the complete stress – strain behavior of diverse metals and alloys and to assessments of the ductile brittle transition, hardness, fatigue, fracture mechanics, shear banding, and strain rate sensitivity properties; and most recently, has led to order-of-magnitude increases in strength being obtained for nanopolycrystalline materials. The H-P parameters, $\sigma_{0\varepsilon}$ and k_{ε}, are based, respectively, in the Taylor (dislocation density) and Griffith (stress concentration) theories of plasticity and fracturing.

OUTLINE

The ductile-to-brittle transition for steel

1. A.H. Cottrell, *Trans. TMS-AIME* ***212***, *192-203 (1958)*

For brittleness, with γ being fracture surface energy

$$k_y(\sigma_{0y}\ell^{1/2} + k_y) \geq CG\gamma$$

in which the yield stress follows a Hall-Petch dependence

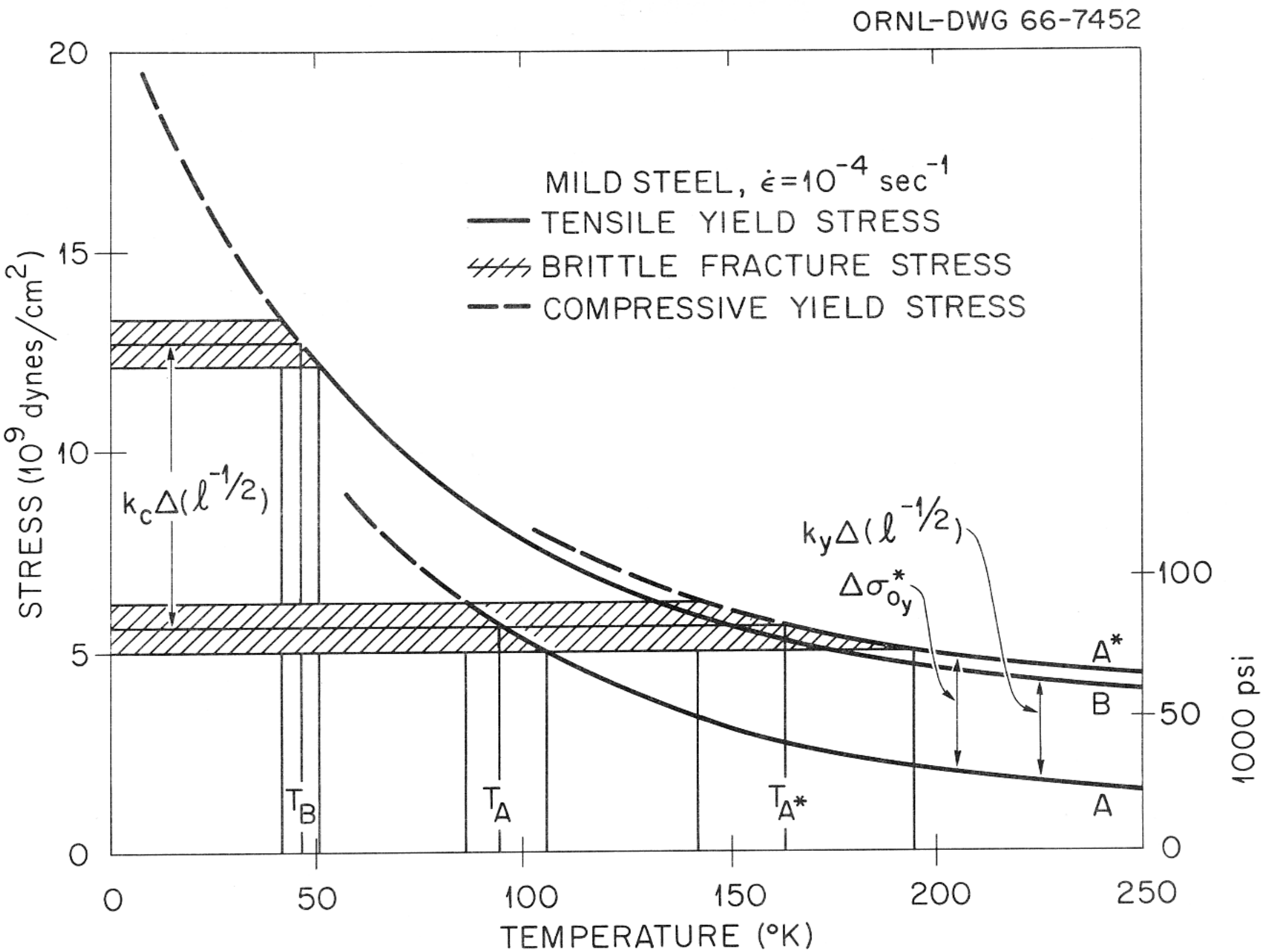
$$\sigma_y = \sigma_{0y} + k_y\ell^{-1/2}$$

2. N.J. Petch, *Philos. Mag.* ***3***, *1089-1097 (1958)*

$$T_C = (1/\beta)[\mathbf{ln}B - \mathbf{ln}\{(CG\gamma/k_f) - k_f\} - \mathbf{ln}\ell^{-1/2}]$$

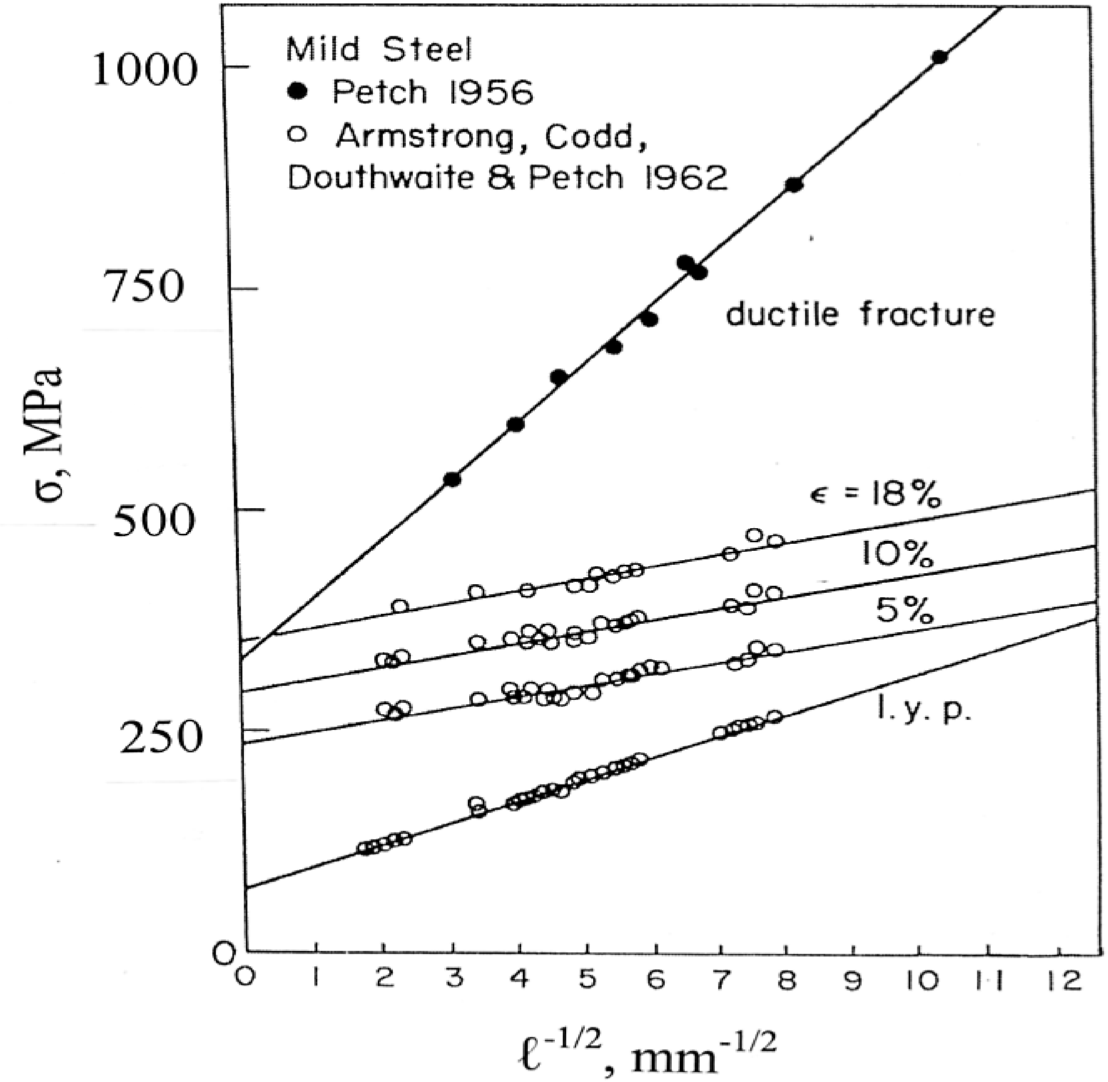
in which (β, B) apply for the temperature dependence of the thermal stress component of σ_{0y} and k_f is the H-P microstructural stress intensity for ductile fracturing

A dbtt determined by σ_y and σ_C measurements



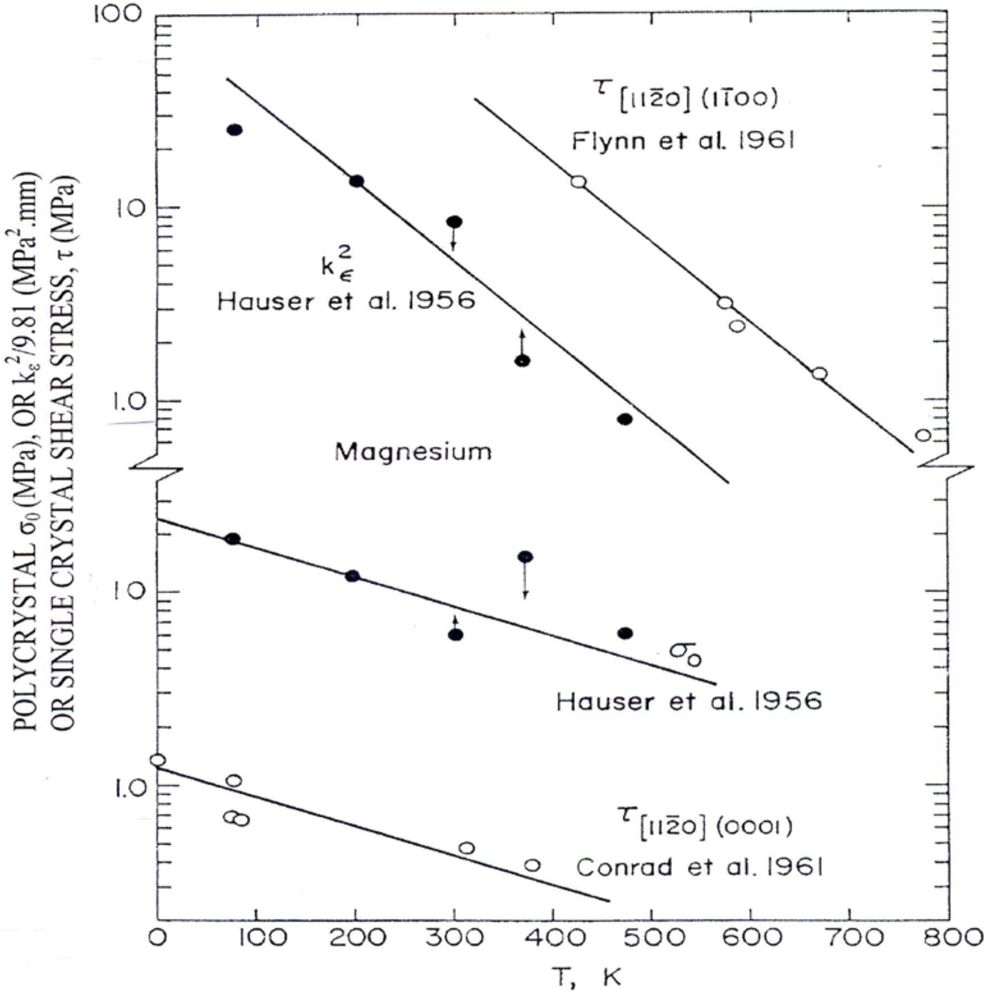
R.W. Armstrong, *Radiation Metall. Sect., Sol. St. Phys. Div., Oak Ridge Nat. Lab., ORNL-4020 (1966); Metall. Trans.* ***1****, 1169-1176 (1970).* Steel **A** has a grain size of 100 µm; **B** has a grain size of 5 µm; and, steel **A*** is the same as A but with $\Delta\sigma_{0y}^*$ = +290 MPa

Hall-Petch for the complete σ – ε behavior



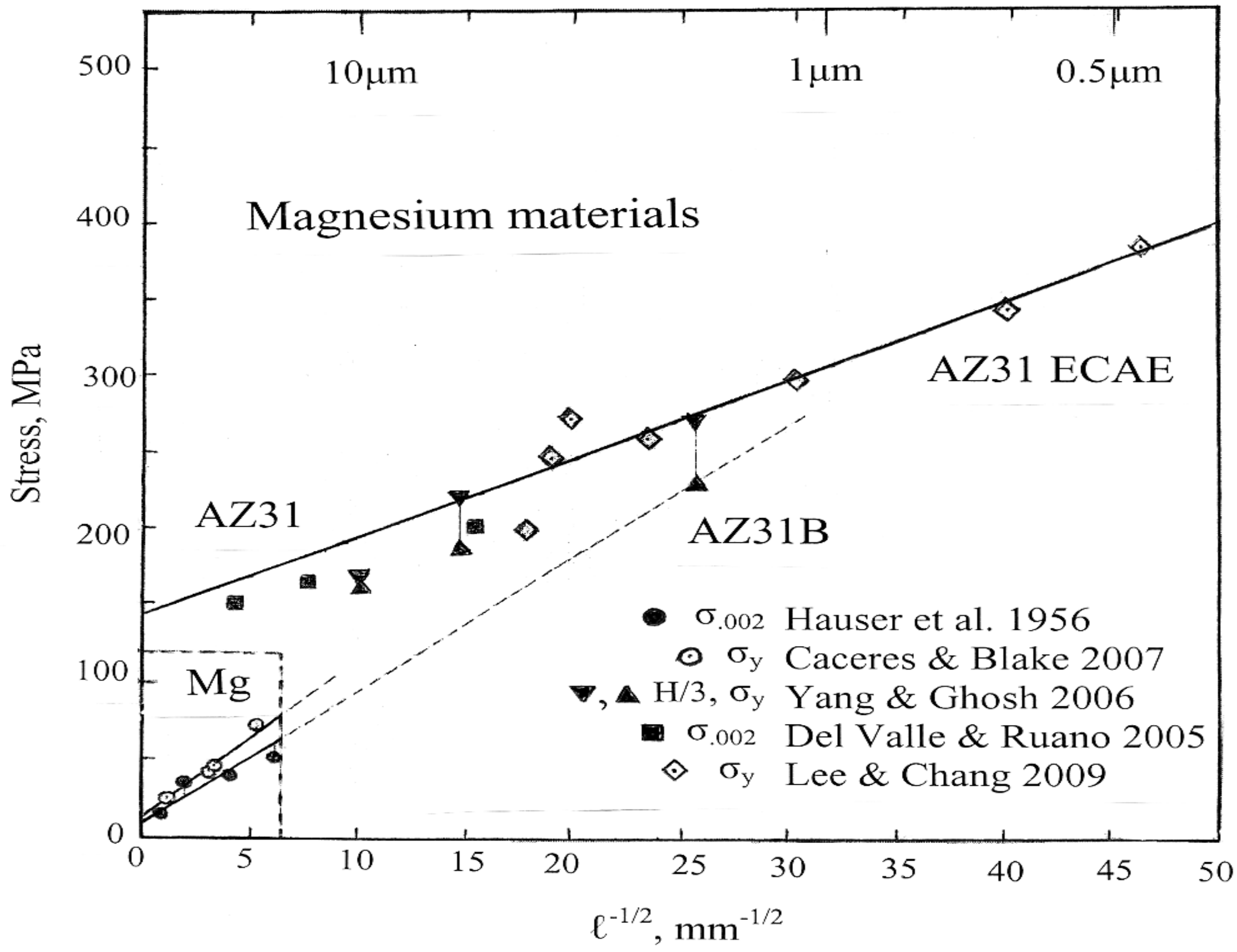
R.W. Armstrong, "The influence of polycrystal grain size on several mechanical properties of materials", *Metall. Trans*. **1**, 1169 -1174 (1970).

Mg: $\sigma_\varepsilon = m[\tau_{0\varepsilon} + k_{S\varepsilon}\ell^{-1/2}]$; $k_{S\varepsilon} = [\pi m^* G b \tau_C / 2\alpha]^{1/2}$



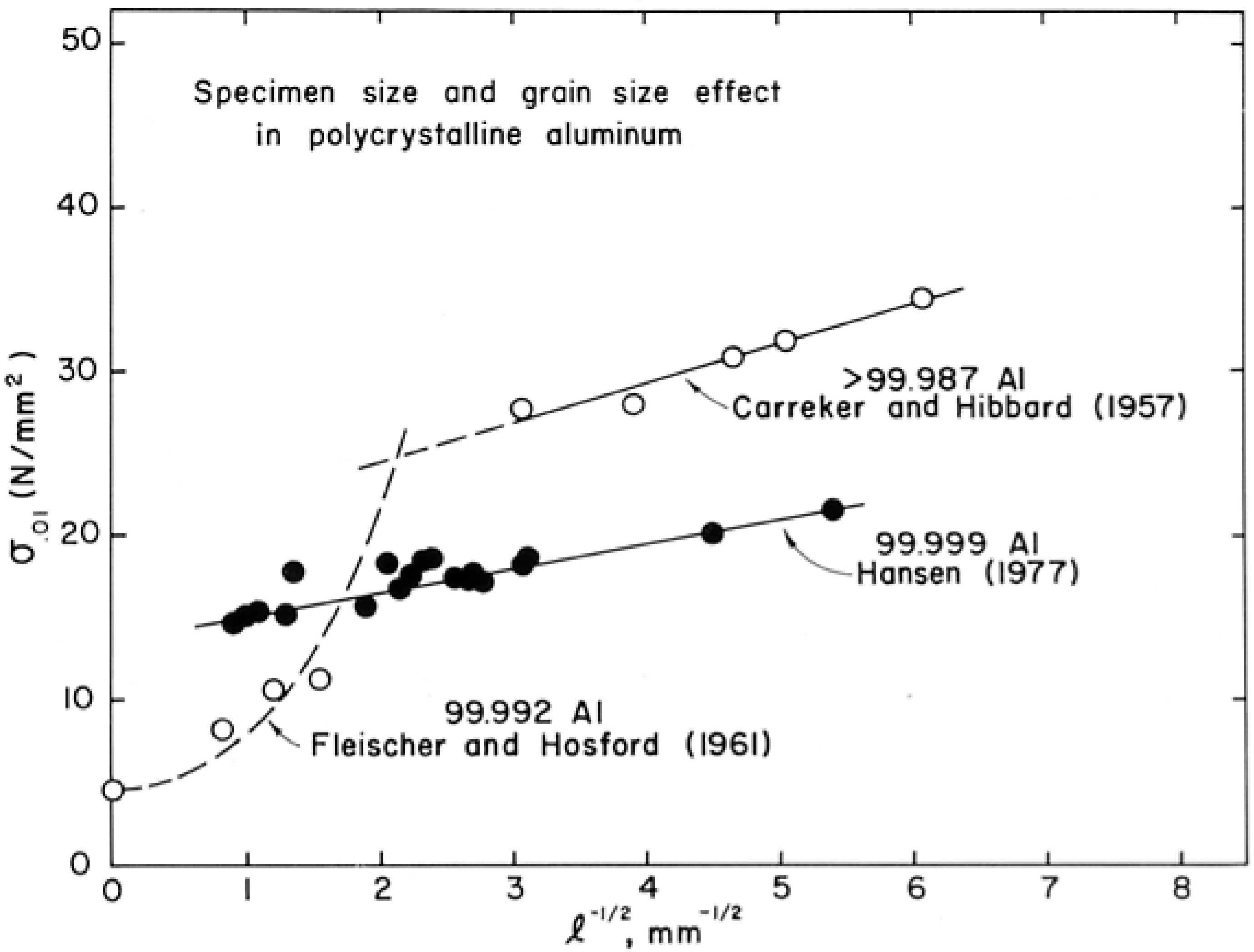
R.W. Armstrong, Theory of the tensile ductile - brittle behavior of polycrystalline hcp materials; with application to beryllium", *Acta Metall*. **16**, 347-355 (1968)

Hall-Petch compilation for Mg and AZ31 alloy materials



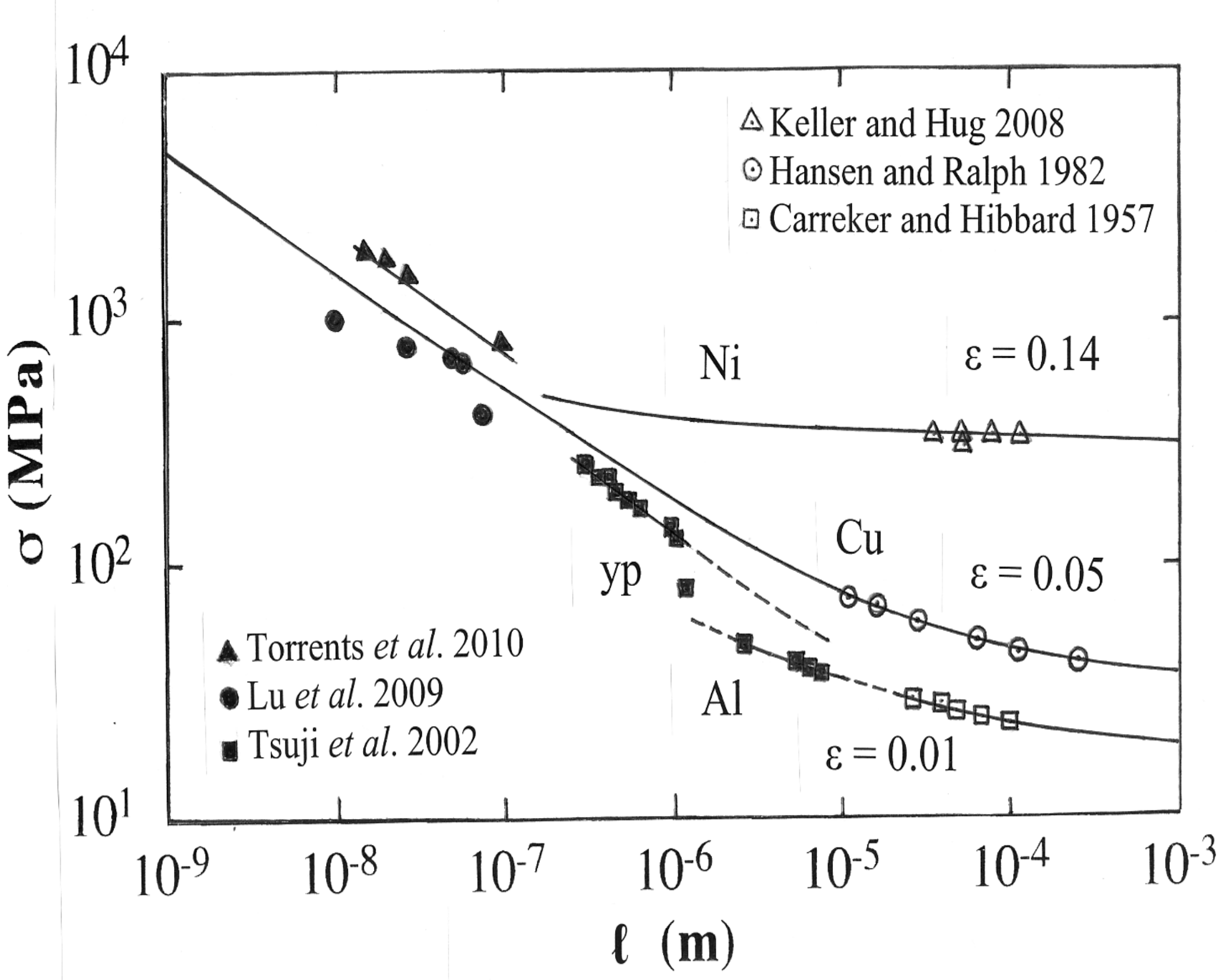
R.W. Armstrong, "Hall-Petch analysis for nanopolycrystals", Proc. 33rd Risoe Int. Symp. Mater. Sci.: *Nanometals – Status and Perspective*, ed. S. Faester *et al*. (Tech. Univ. Denmark, Risoe Campus, Roskilde, 2012) pp. 181-199.

Specimen size/grain size/solute effects in Al



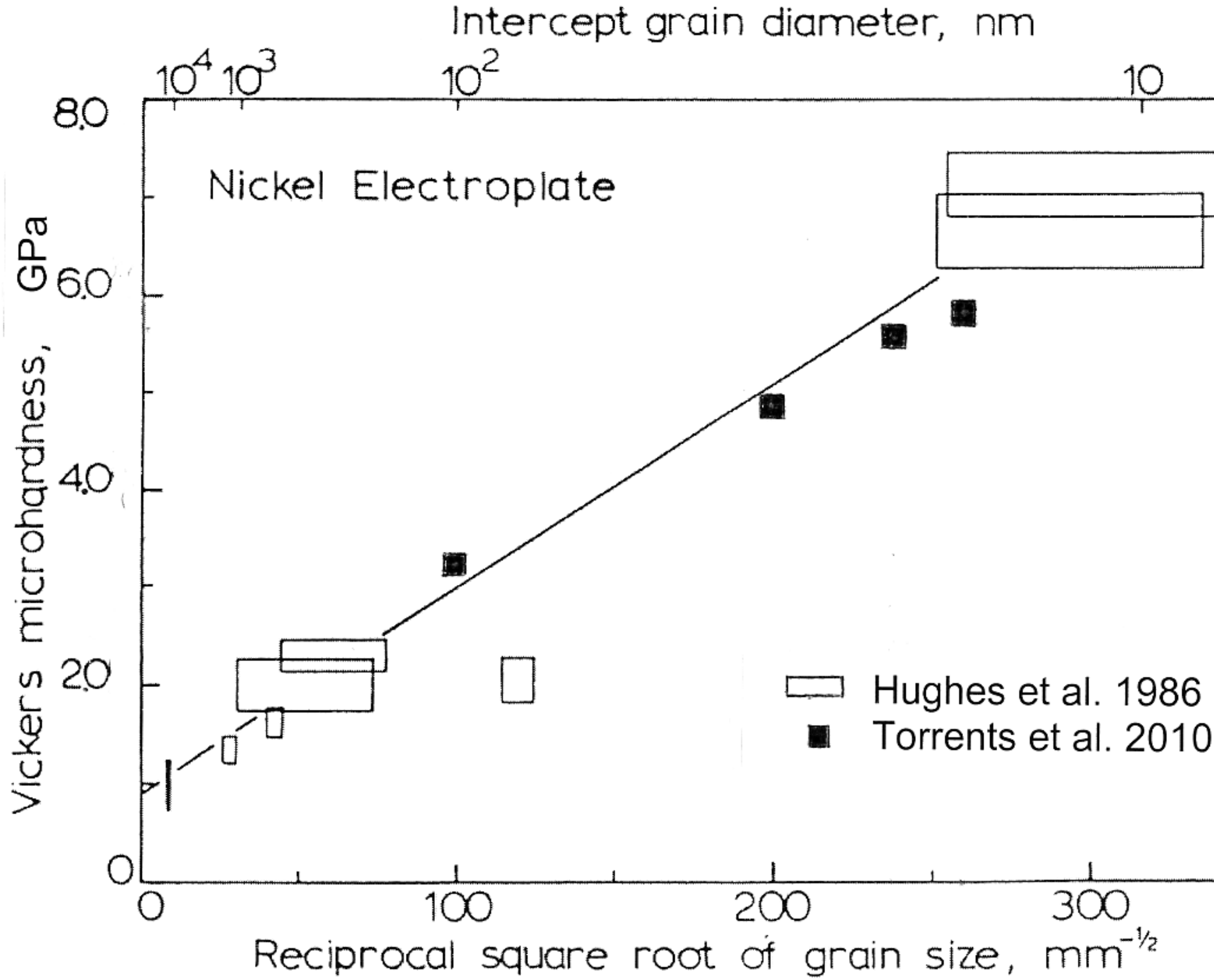
R.W. Armstrong, in *Physics of Materials: A Festschrift for Dr. Walter Boas on the Occasion of his 75th Birthday*, ed. by D.W. Borland *et al*. (CSIRO and Univ. Melbourne Press, Australia, 1979) 1-11.

Conventional to nano-scale fcc H-P results



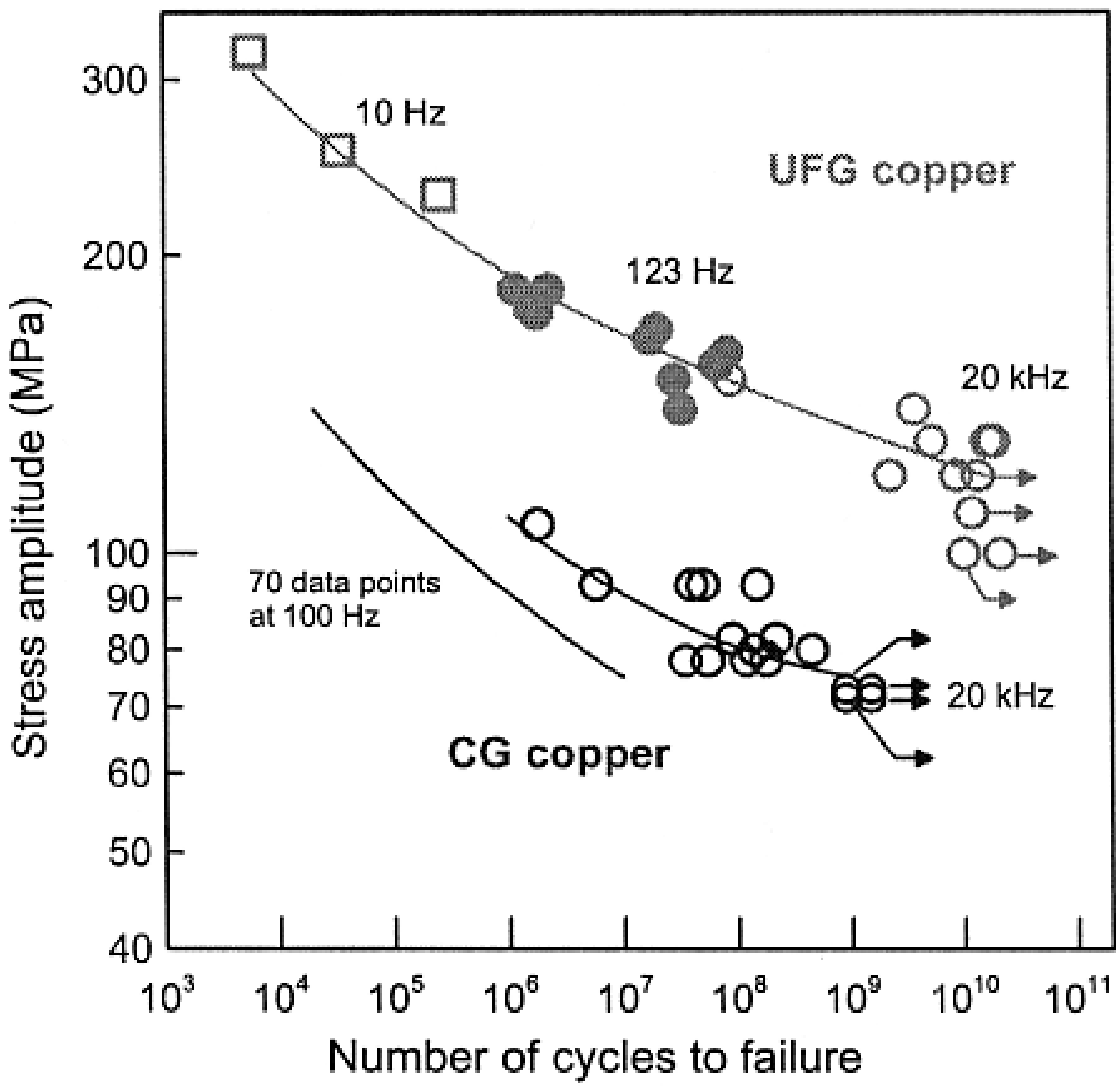
R.W. Armstrong, "60 years of Hall-Petch: Past to present nano-scale connections", *Strength of Fine Grained Materials,* Univ. Tokyo, 7/16-18/2013

Hall-Petch results for the hardness of nickel



E.O. Hall, Nature, **173**, 948-949 (1954); R.W. Armstrong, W.L. Elban and S.M. Walley, Int. J. Mod. Phys. B, **27**, [8], 1330004 (2013).

Fatigue of coarse and ultrafine (300 nm) grained copper



P. Lukas, L. Kunz, L. Navratilova and O. Bokuvka. *Mater. Sci. Eng. A*, **528**, 7036-7040 (2011); S.D. Antolovich and R.W. Armstrong, *Prog. Mater. Sci*., in print (2013).

Thermal activation – strain rate analysis (TASRA)

$$(d\varepsilon/dt) = (1/m)\rho b v$$

dislocation velocity: $v = v_0 \mathbf{exp}[-(G_0 - \int A^* b\, d\tau_{Th})/kT]$

in which $\tau_{Th} = \tau - \tau_G$

activation area: $A^* = (kT/b)[\partial \ln(d\varepsilon/dt)/\partial \tau_{Th}]_T$ *and* $A^*b = W_0/\tau_{Th}$

Computational (Z-A) equations:

$$\sigma = \sigma_G + B\mathbf{exp}[-\beta T] +$$

$$B_0[\varepsilon_r(1 - \mathbf{exp}\{-\varepsilon/\varepsilon_r\})]^{1/2}\mathbf{exp}[-\alpha T] + k_\varepsilon \ell^{-1/2}$$

in which

$$(\beta,\alpha) = (\beta_0,\alpha_0) - (\beta_1,\alpha_1)\mathbf{ln}(d\varepsilon/dt)$$

bcc case: $\alpha = \alpha_0 = \alpha_1 = 0$ *fcc case*: $B = 0;\ \beta = \beta_0 = \beta_1 = 0$

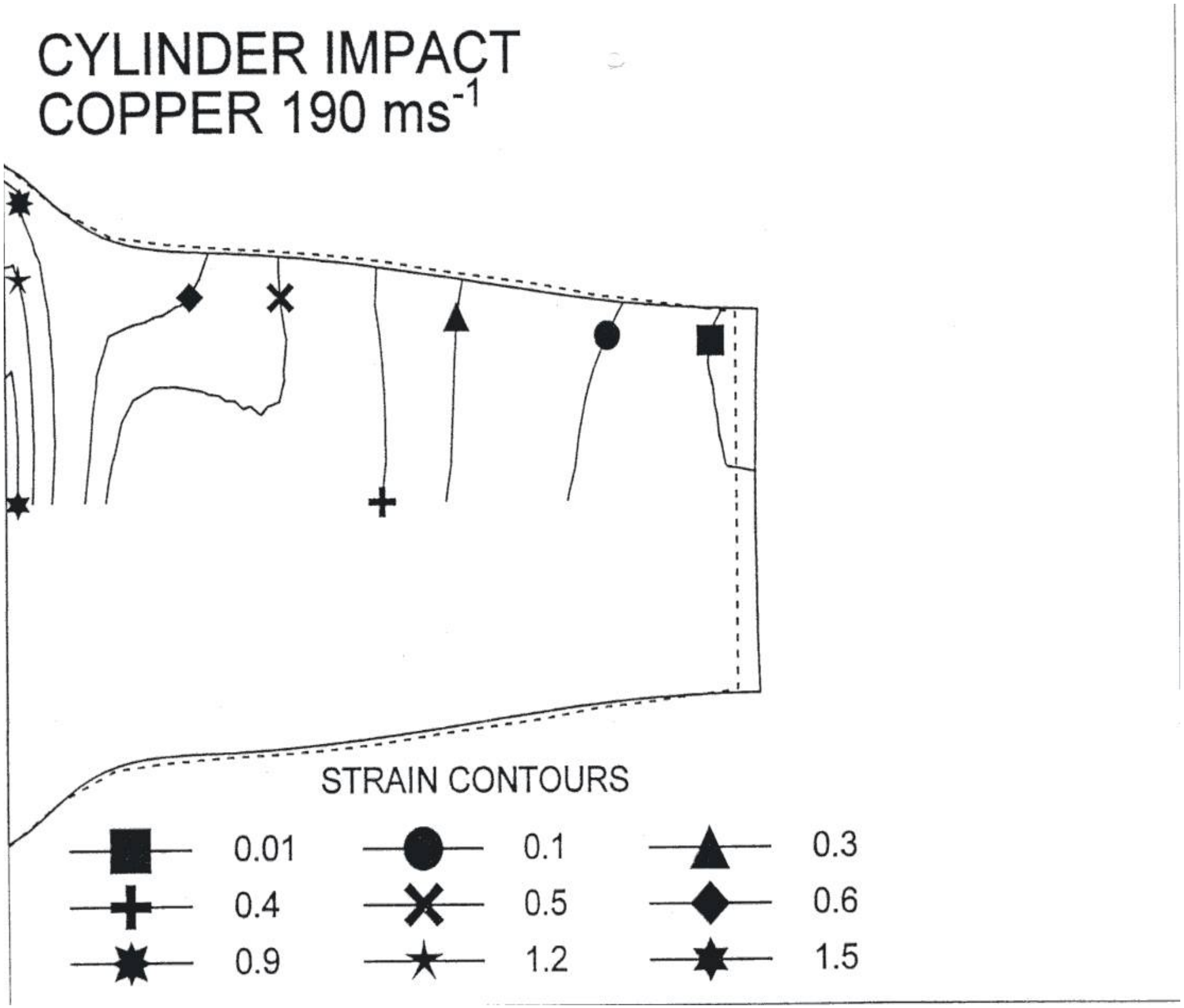
Thus thermal activation is in the yield stress for bcc metals and alloys and is in the strain hardening for fcc metals and alloys.

F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.* **61**, 1816-1825 (1987)

F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.* **68**, 1580-1591 (1990)

F.J. Zerilli, *Metall. Mater. Trans. A*, **35A**, 2547-2555 (2004)

Z-A equation application to copper cylinder impact



Taylor Cylinder Impact Test: $0.01 < \varepsilon < \sim 1.5$; $0 < (d\varepsilon/dt) < \sim 10^5\ s^{-1}$; $300 < T < \sim 600$ K

F.J. Zerilli and R.W. Armstrong, *J. Appl. Phys.*, **61**, 1816-1825 (1987)

Hall-Petch connection with fracture mechanics

1. Bilby, Cottrell and Swinden (1963):

$$(s/c) = [\sec(\pi\sigma_F/2\sigma_y)] - 1$$

in which *s* is length of plastic zone, *c* is crack half-length, σ_F is fracture stress and σ_y is yield stress.

2. Griffith-form by Armstrong (1973); with $\sigma_y \rightarrow \sigma_{F0}$, crack-free fracture stress:

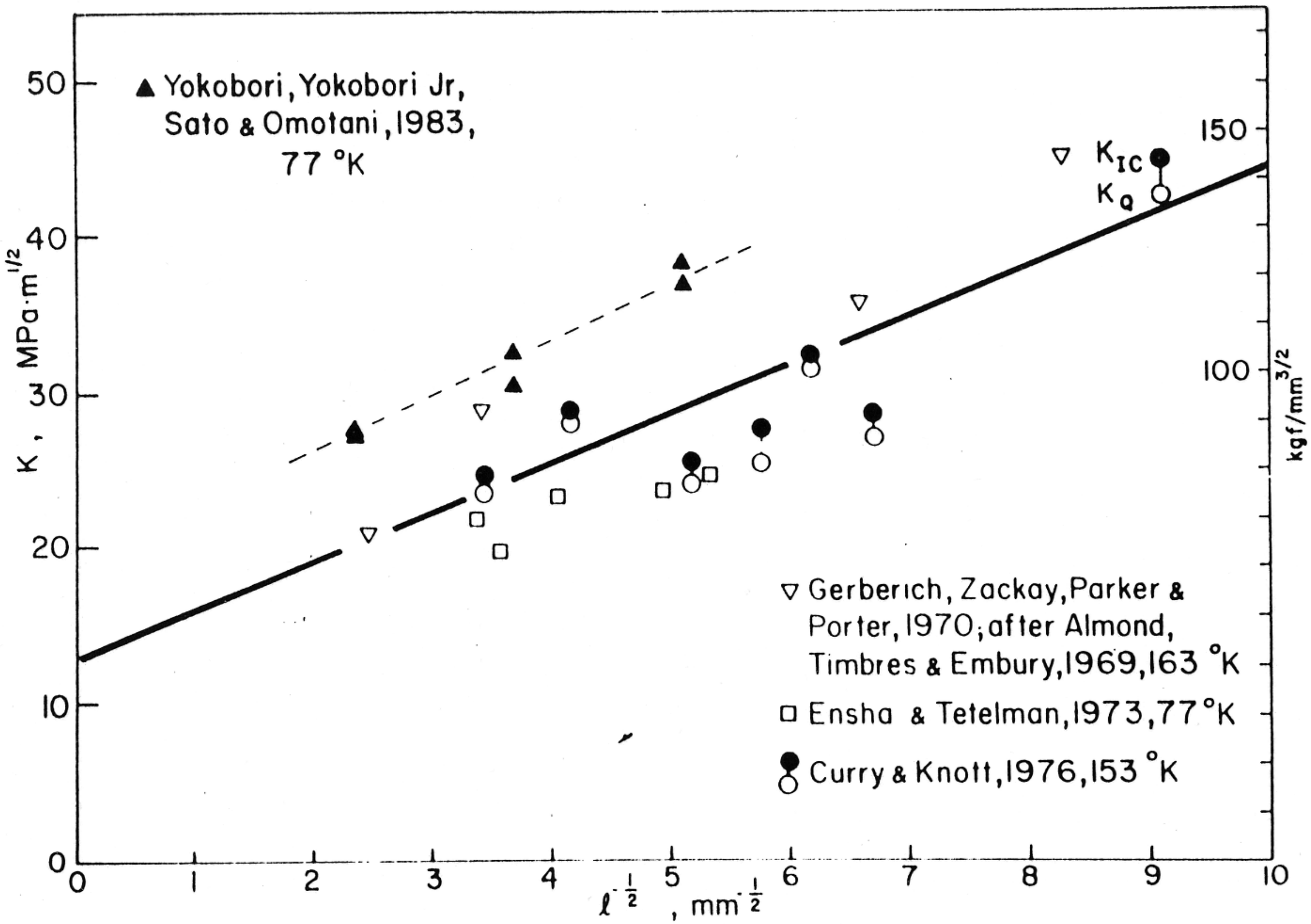
$$\sigma_F \approx A\sigma_{F0}\,[s/(c + s)]^{1/2}$$

3. Fracture mechanics*: $\sigma_F = K_{Ic}/(\pi c)^{1/2}$

$$K_{Ic} = (8/3\pi)^{1/2}[\sigma_{0C} + k_C \ell^{-1/2}]s^{1/2}$$

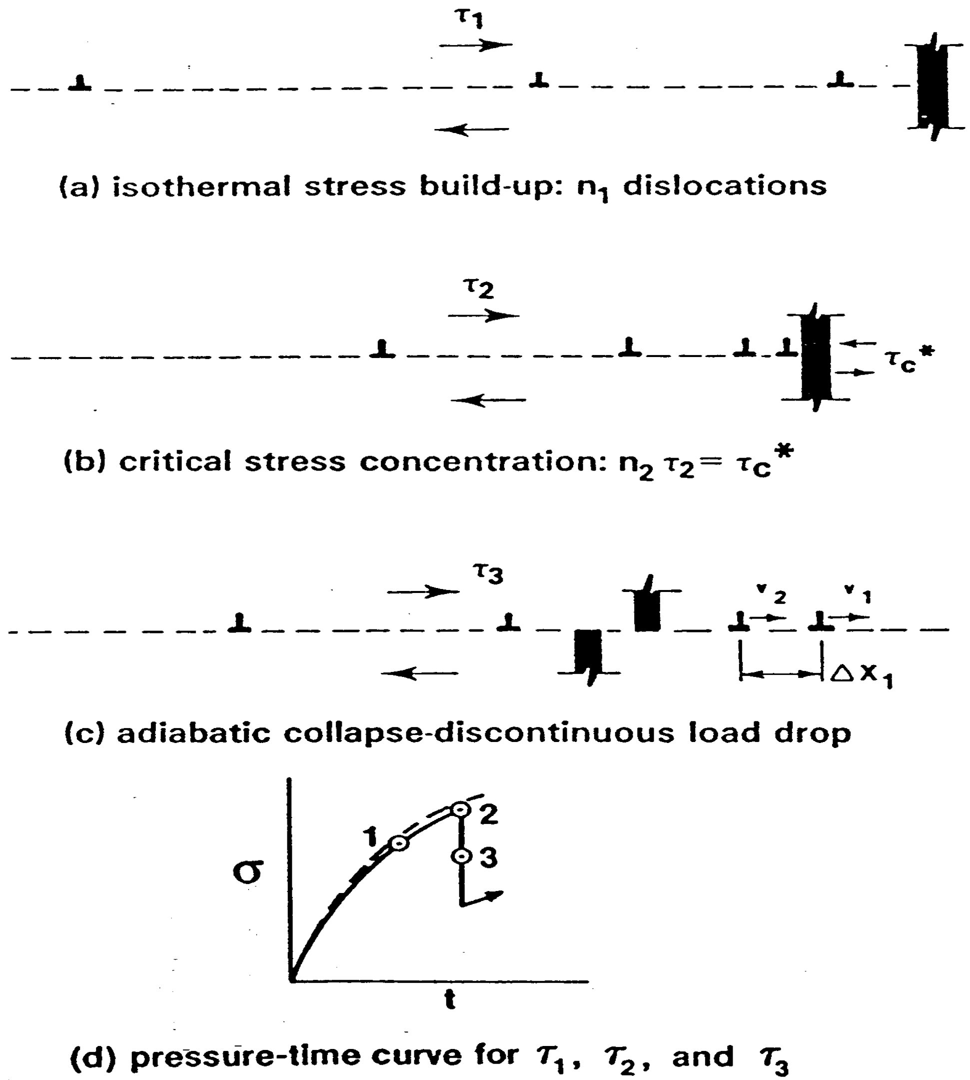
G.R. Irwin, *7th Sagamore Ordnance Materials Research Conference* (Syracuse Univ. Res. Inst., N.Y., 1961) p. 63; *R.W. Armstrong, *Eng. Fract. Mech*., **77**, 1348 (2010).

Compilation of H-P type results for *K*



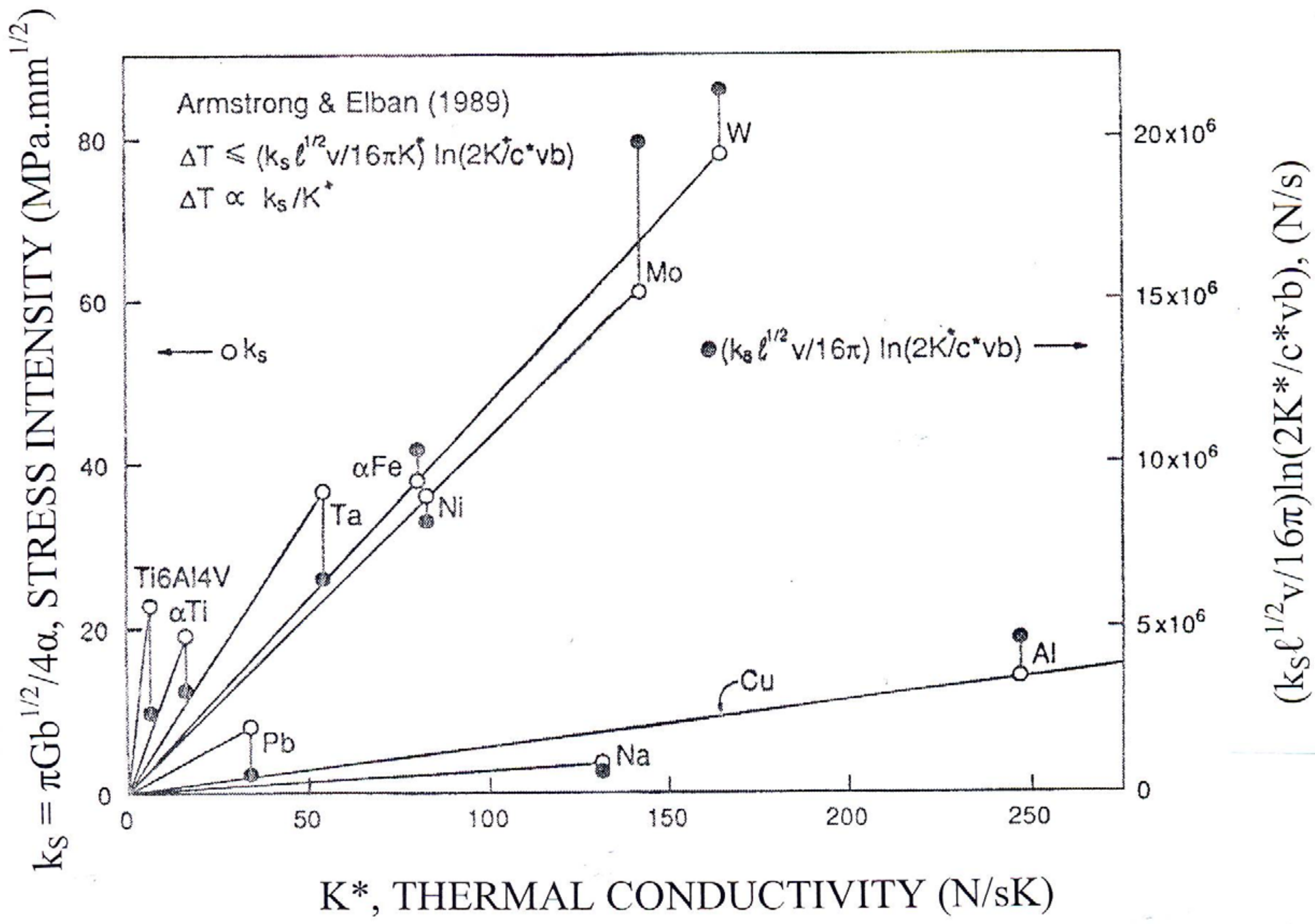
R.W. Armstrong, in "A Special Issue in Honor of Professor Takeo Yokobori on the Occasion of His 70th Birthday", *Eng. Fract. Mech.*, **28**, [5/6], 529-538 (1987).

Dislocation pile-up model for shear banding



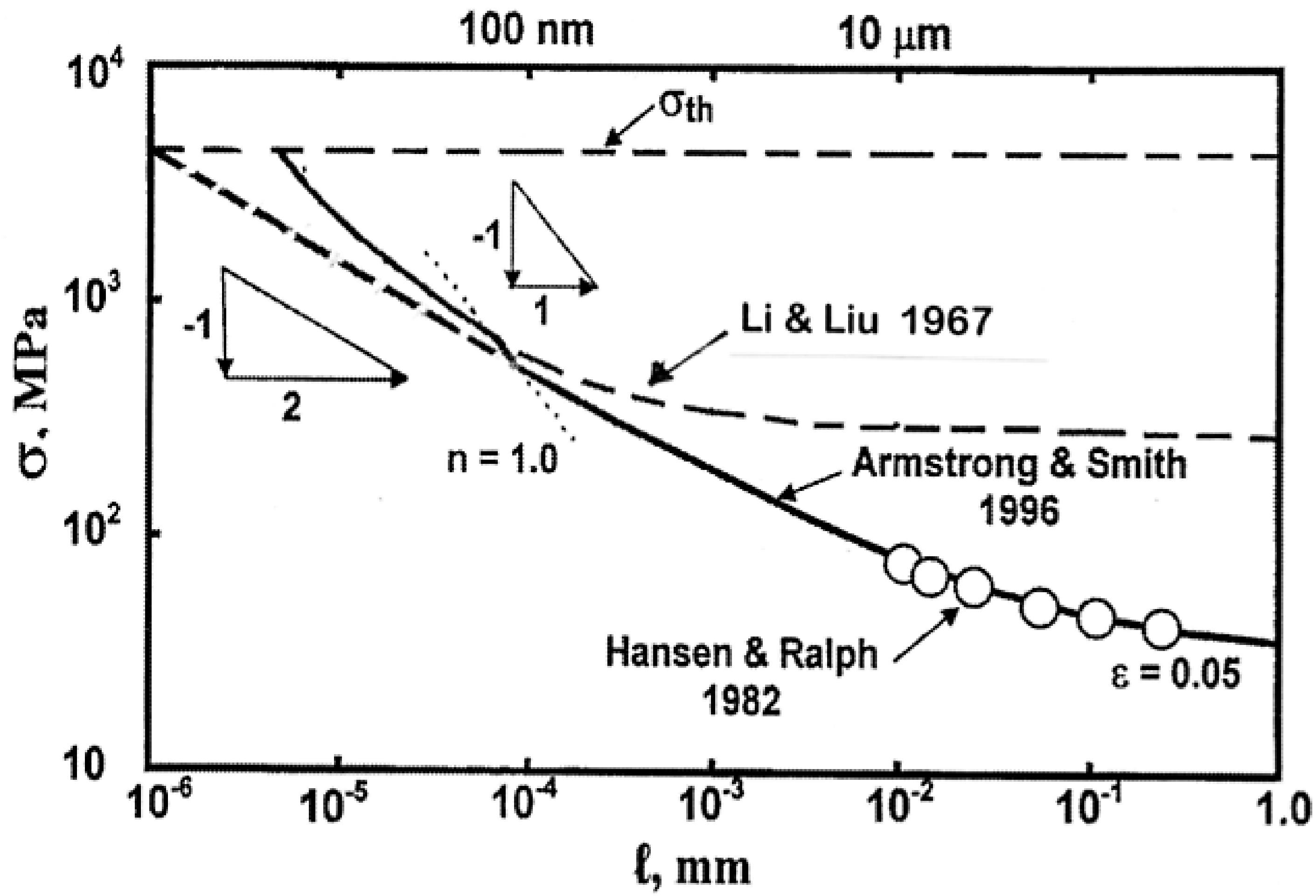
R.W. Armstrong, C.S. Coffey and W.L. Elban, "Adiabatic heating at a dislocation pile-up avalanche", *Acta Metall*., **30**, 2111-2118 (1982)

Susceptibility factor (k_S/K^*) for shear banding



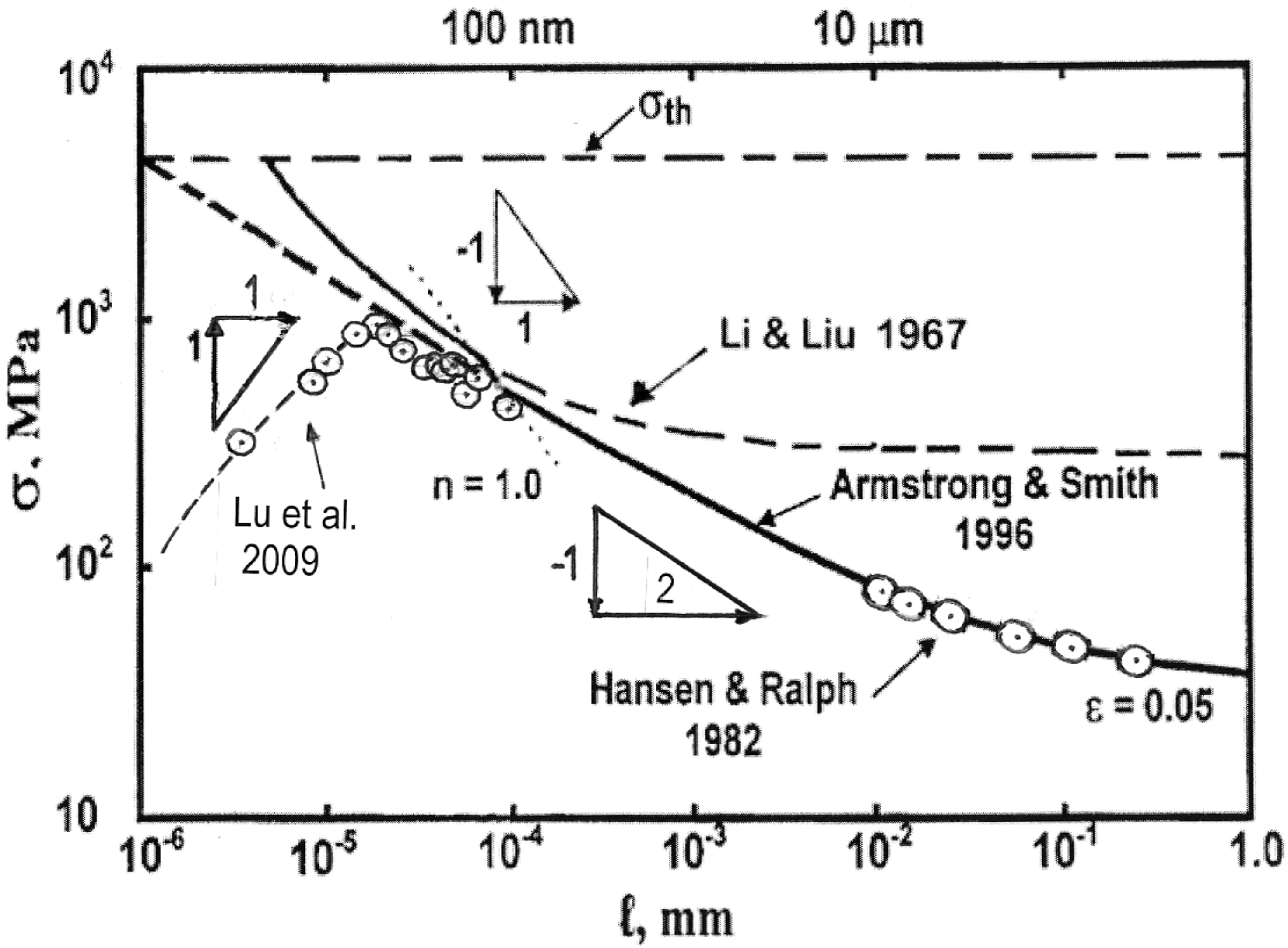
R.W. Armstrong and W.L. Elban, *Mater.Sci. Eng.* **A122**, L1-L3 (1989)

Transition to a single dislocation loop expanding against the grain boundary resistance in copper



R.W. Armstrong and T.R. Smith, "Dislocation pile-up predictions for the strength properties of ultrafine grain size fcc metals", in *Processing and Properties of Nanocrystalline Materials*, ed. by C. Suryanarayana et al. (TMS-AIME) 345-354.

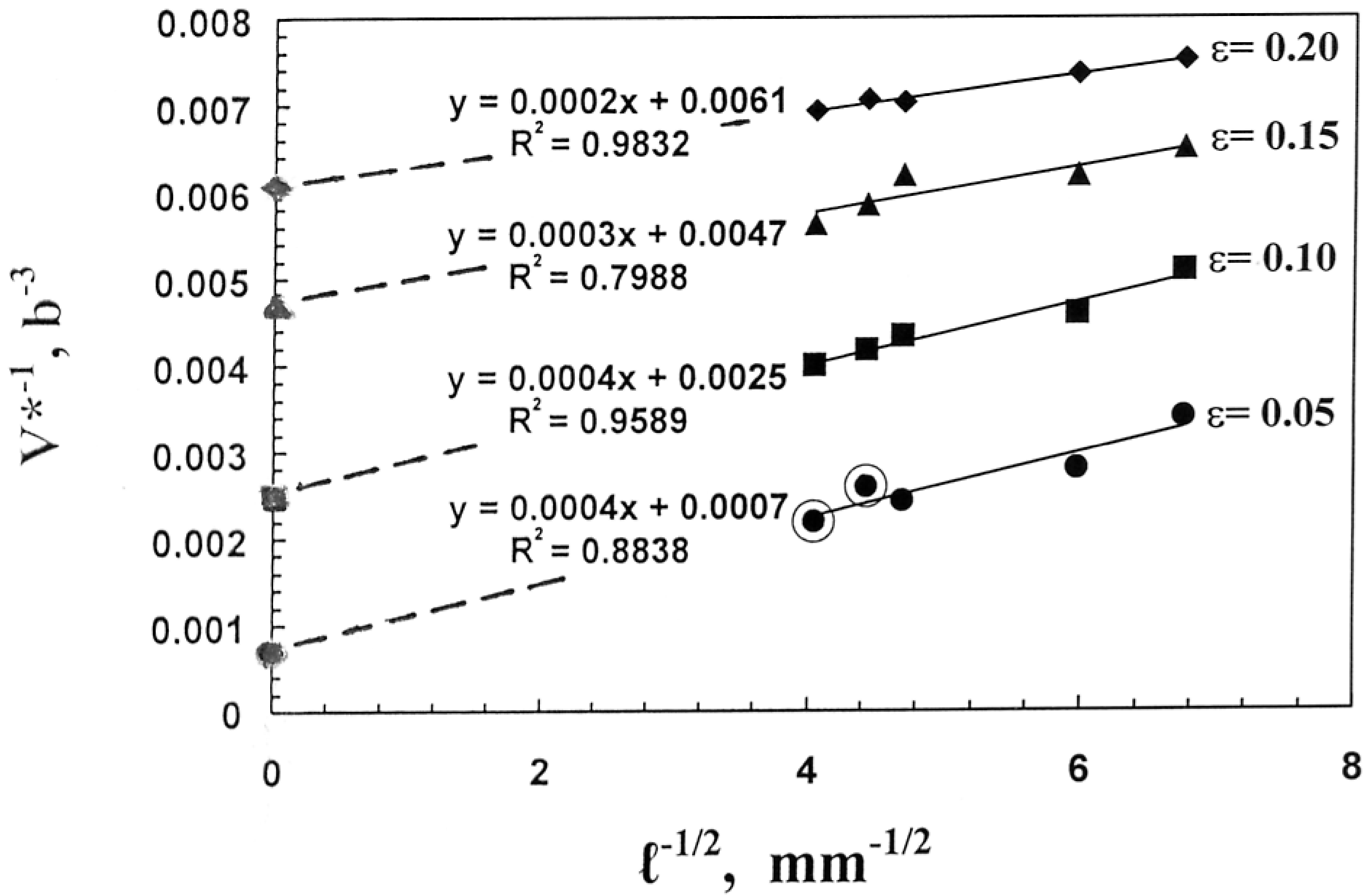
Application to nano-twinned Cu



L.Lu, X, Chen, X. Huang and K. Lu, *Science*, **323**, 607-610 (2009); R.W. Armstrong, in *Strength of Fine Grained Materials*, The University of Tokyo, 7/16-18/2013.

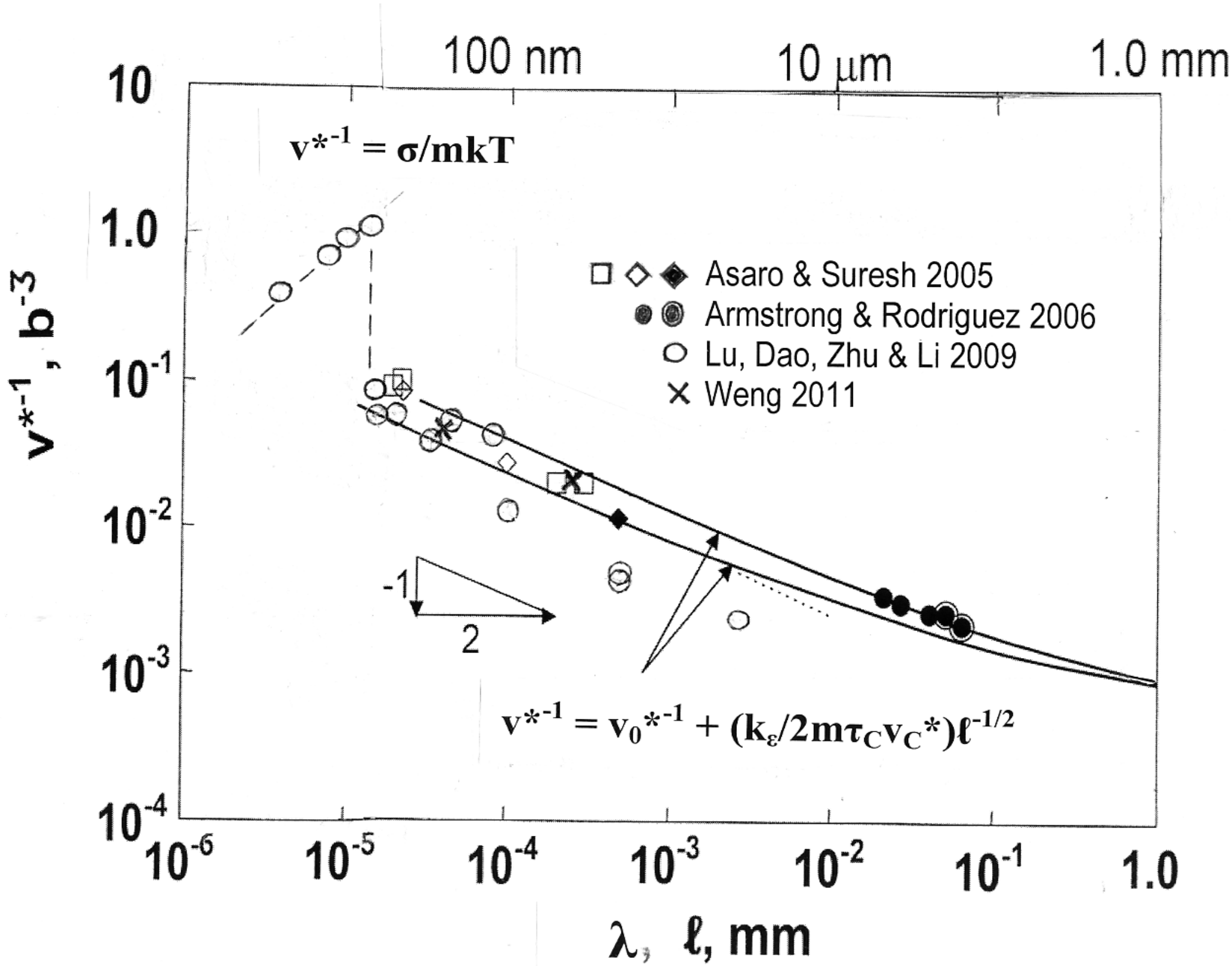
Cross-slip controls k_ε for nickel

$$v^{*-1} = v_0^{*-1} + (k_\varepsilon/2m_T\tau_C v_C^*)\ell^{-1/2}$$



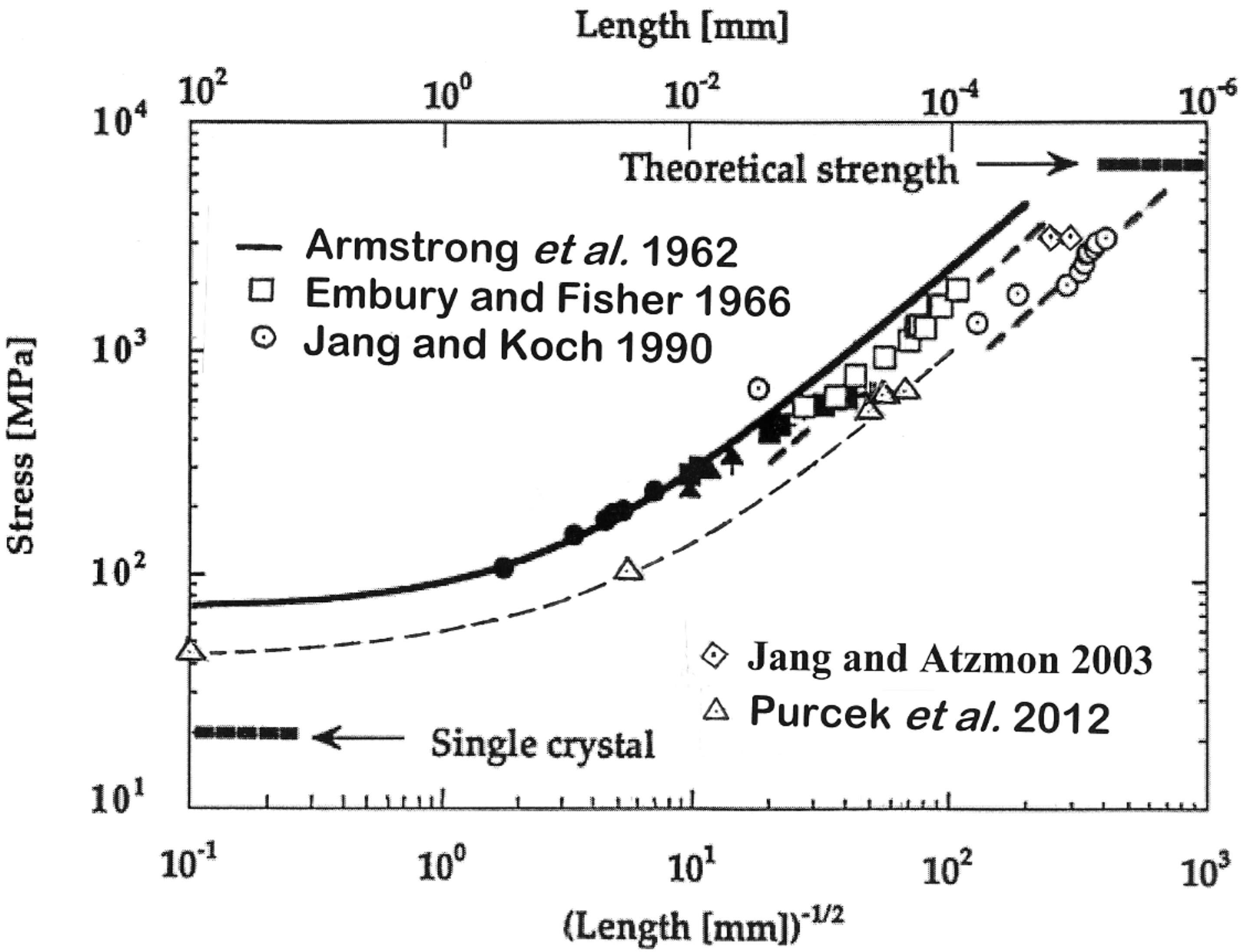
T. Narutani and J. Takamura, *Acta Metall. Mater*., **39**, 2037-2049(1991); P. Rodriguez , *Metall. Mater. Trans. A*, **35A**, 2697-2705 (2004); R.W. Armstrong and P. Rodriguez, *Philos. Mag*., **86**, 5787-5796 (2006).

H-P type v^{*-1} measurements for Cu and Ni



R.W. Armstrong, in *Strength of Fine Grained Materials – 60 years of Hall-Petch*, The University of Tokyo, 7/16/2013

Grain boundary disorder in fine grained steel?



R.W. Armstrong, in *Strength of Fine Grained Materials – 60 years of Hall-Petch*, The University of Tokyo, 7/16-18/2013

SUMMARY

1. Hall and Petch really started something!

$$\sigma_i = \sigma_{0i} + k_i \ell^{-1/2}; \; i = y, C$$

2. Petch and colleague extensions

$$\sigma_\varepsilon = m_T[\tau_{0\varepsilon} + k_{S\varepsilon}\ell^{-1/2}]$$

3. Evaluation of the stress intensity

$$k_\varepsilon = m_T[\pi m^* G b \tau_C / 2\alpha]^{1/2}$$

4. From the TASRA for [dε/dt]

$$v^{*-1} = v_{0\varepsilon}^{*-1} + [k_\varepsilon / 2 m_T \tau_C v_C^*]\ell^{-1/2}$$

5. DBTT, σ – ε, hardness, fatigue, FM, shear banding, and fine grained material results