Sir Alan Cottrell and the Dislocation Mechanics of Fracturing

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Sir Alan Cottrell led the transformation of physical metallurgy from an observational science to analytical description.

TOPICS 1

The ductile-to-brittle transition temperature

The Cottrell-Petch theory of onset of bcc metal brittleness

1. A.H. Cottrell (1958)

For brittleness, with γ being fracture surface energy , C numerical constant, and G shear modulus

$$k_y(\sigma_{0y}\ell^{1/2} + k_y) \geq CG\gamma$$

in which the yield stress follows a Hall-Petch dependence

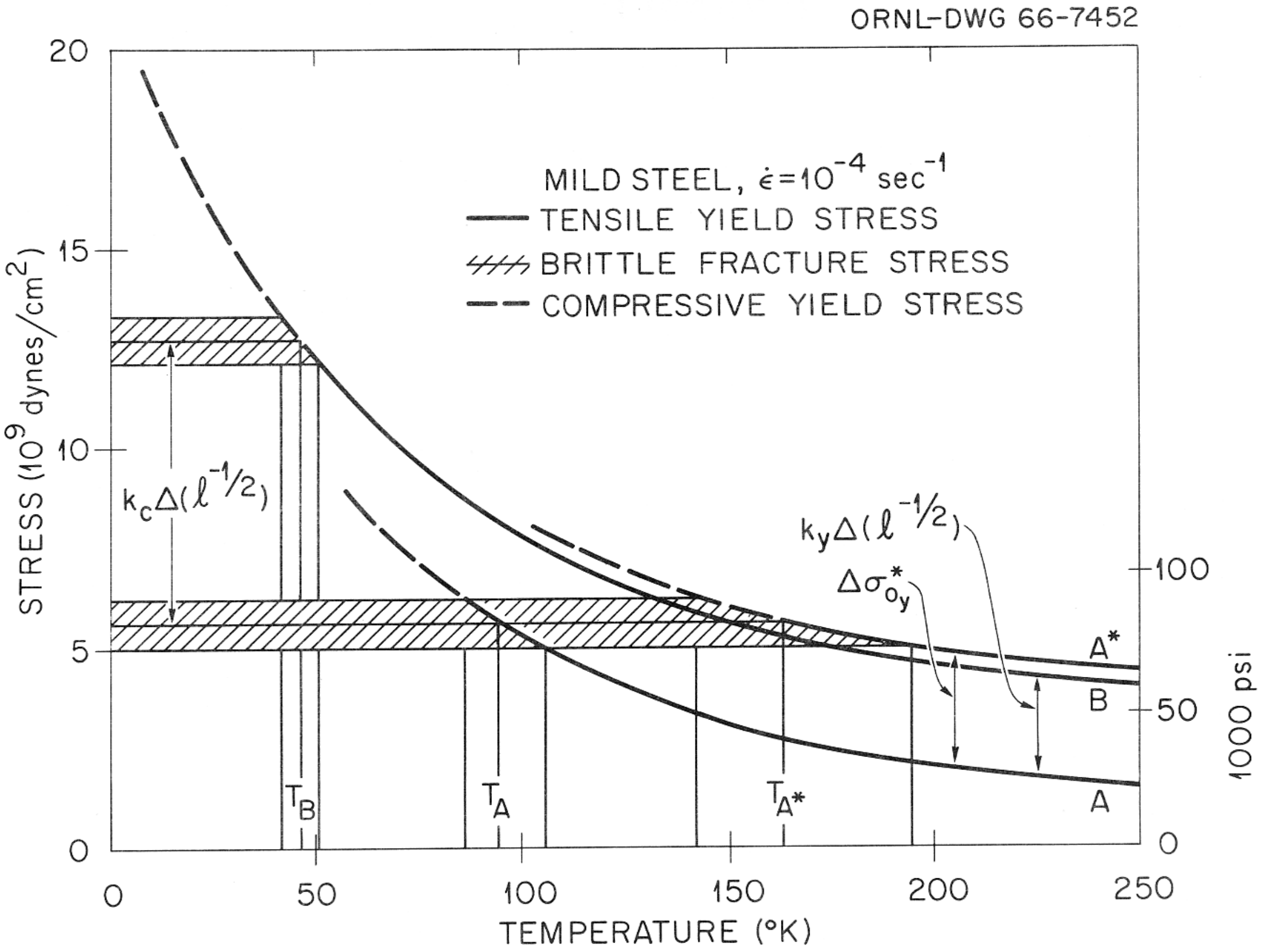
$$\sigma_y = \sigma_{0y} + k_y\ell^{-1/2}$$

2. N.J. Petch (1958)

$$T_C = (1/\beta)[\mathbf{ln}B - \mathbf{ln}\{(CG\gamma/k_f) - k_f\} - \mathbf{ln}\ell^{-1/2}]$$

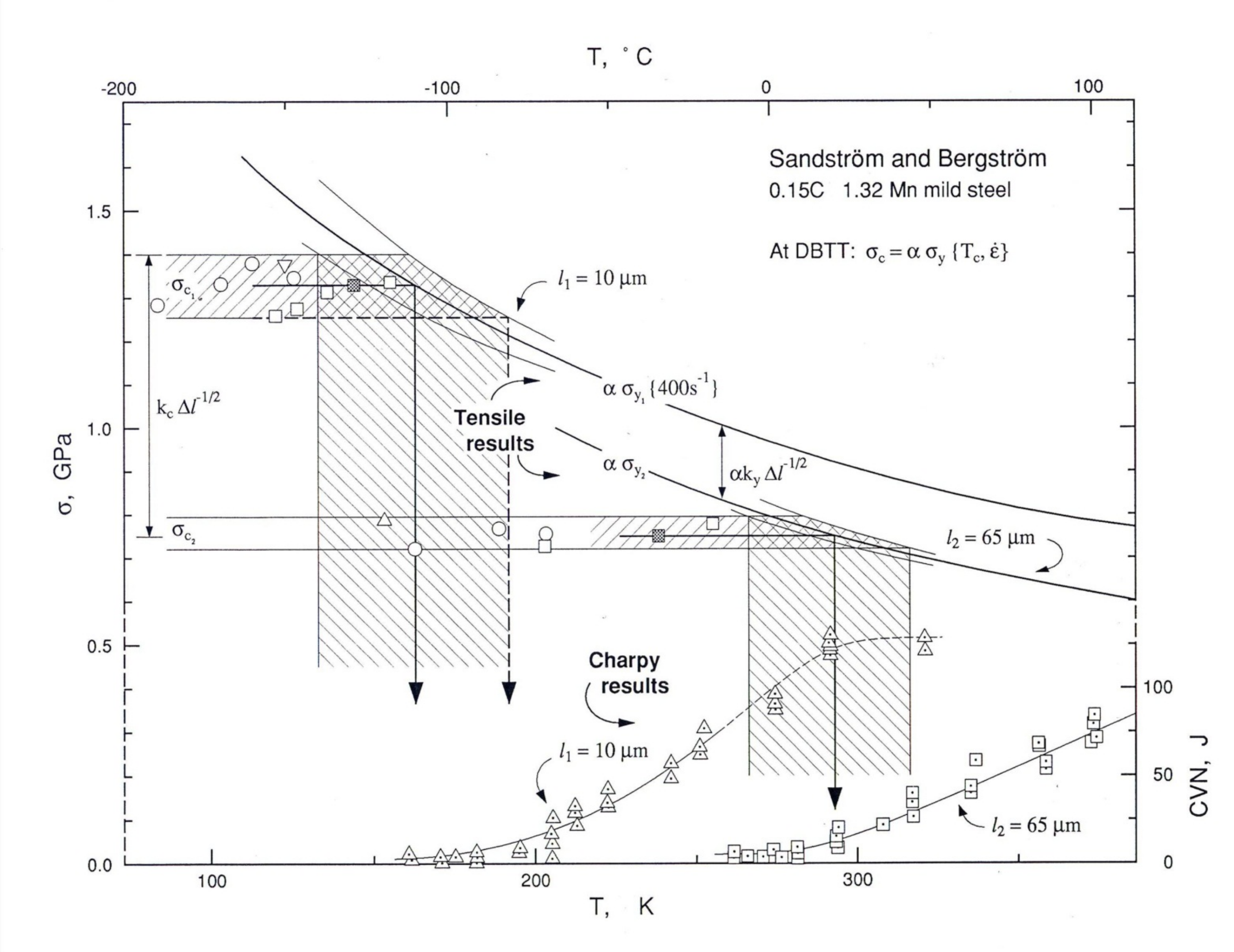
in which (β, B) apply for the temperature dependence of the thermal stress component of σ_{0y} and k_f is the H-P microstructural stress intensity for ductile fracturing

Tensile ductile-brittle transition temperatures for two grain sizes, A =100 μm and B = 5 μm; and, of A altered by neutron irradiation to A* with $\Delta\sigma_{oy}^*$ = 290 MPa.



R.W. Armstrong, Stress – Grain Size Analysis of the Ductile – Brittle Transition for Steel and Similar Metals, *Radiation Metallurgy Section, Solid State Division Progress Report, Oak Ridge National Laboratory*, ORNL-4020 (July 1966).

Correlation of tensile and Charpy ductile-brittle transition results

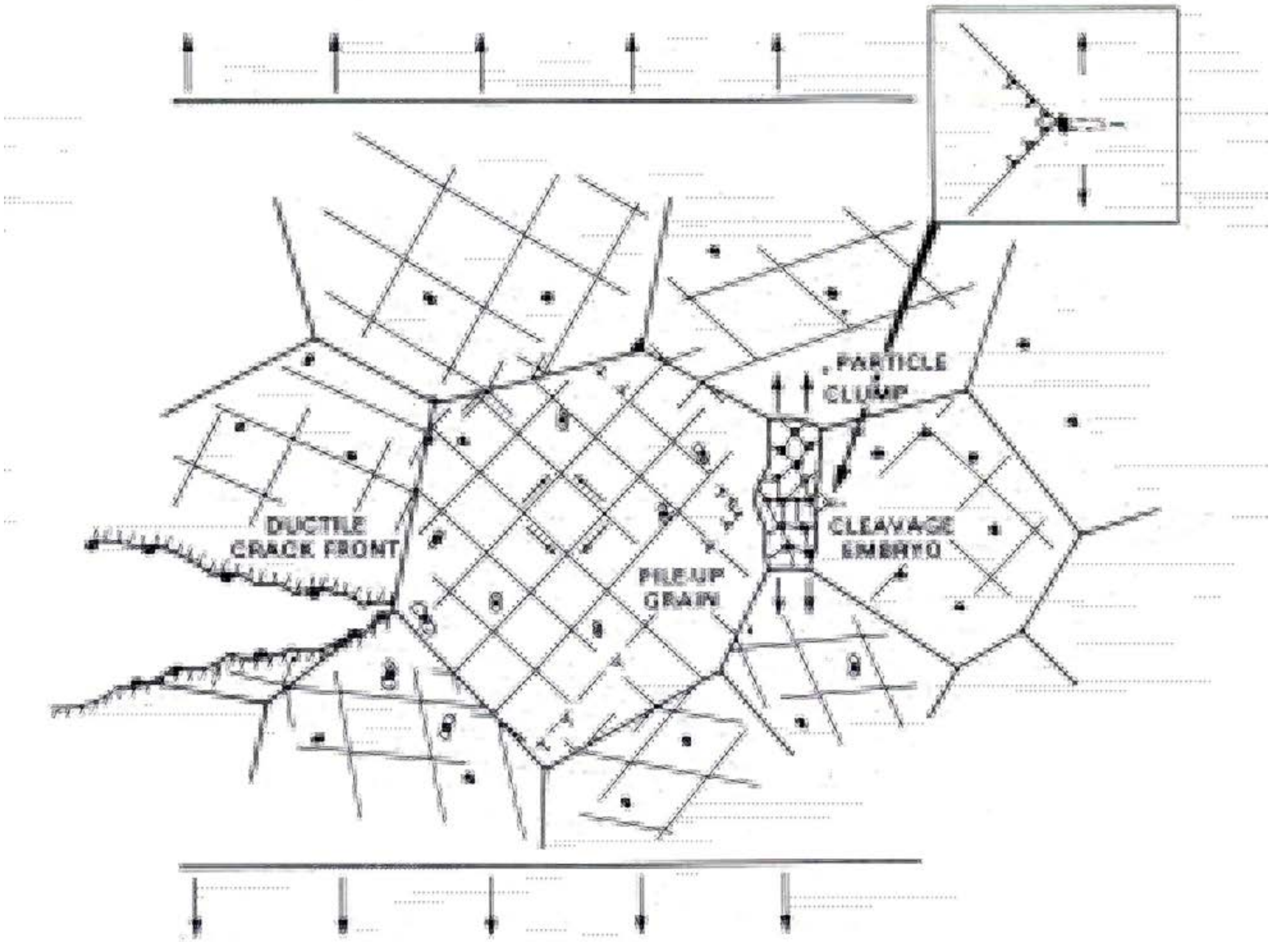


R.W. Armstrong, G.R. Irwin and X.J. Zhang, in *Cleavage Fracture; George R. Irwin Symposium* (TMS-AIME, Warrendale, PA, 1998) pp. 51-58; adapting results of R. Sandstrom and Y. Bergstrom, *Met. Sci.*, **18**, 177-186 (1984).

TOPICS 2

Crack-forming dislocation reactions

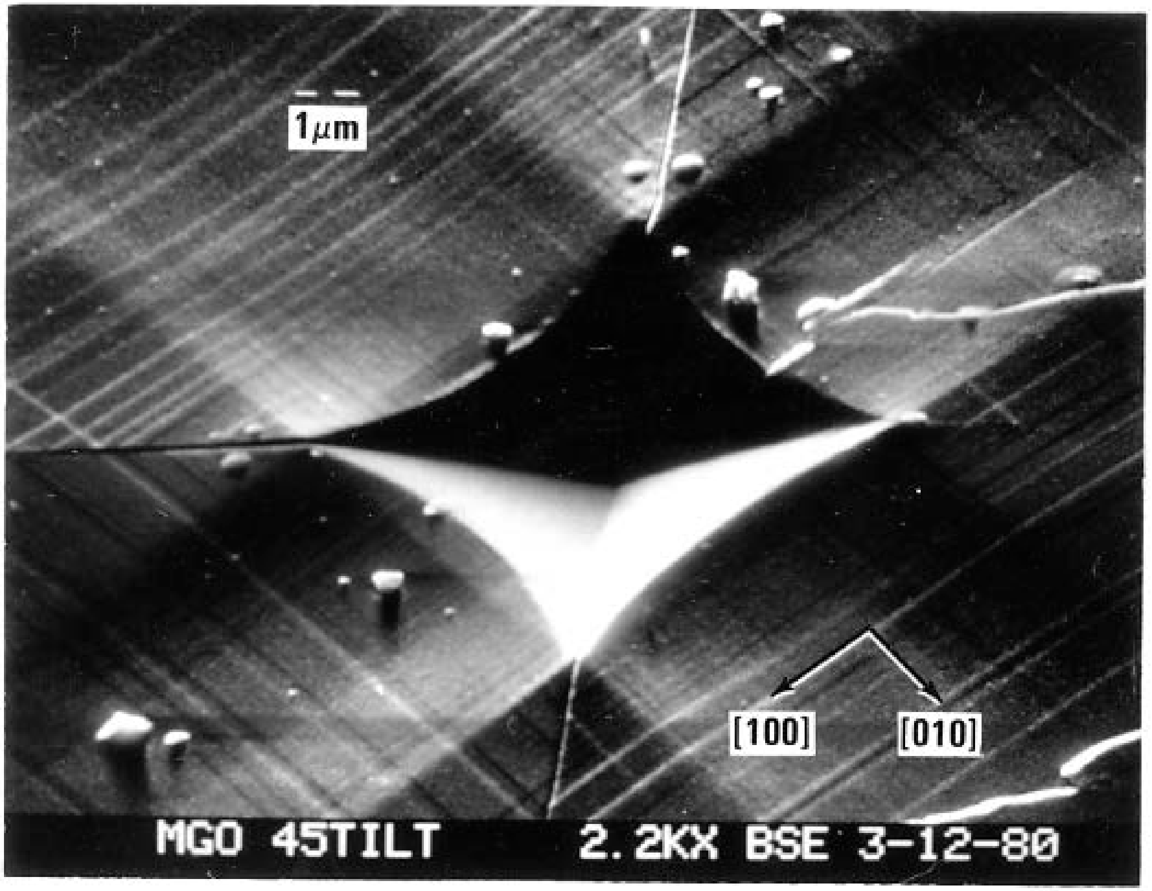
[Ductile tearing] to [H-P grain size and particle clump] to [Cottrell crack reaction] for ductile fracturing to cleavage transition



For [001] T.A.: $(a/2)[11\text{-}1]_{(101)} + (a/2)[\text{-}1\text{-}1\text{-}1]_{(\text{-}101)} = a[00\text{-}1]_{(001)}$

J.P. Gudas, G.R. Irwin, R.W. Armstrong and X.J. Zhang, in: Defect Assessment in components – fundamentals and applications, eds. J.G. Blauel and K.-H Schwalbe ESIS/EGF9 (Mech. Eng. Publ. Ltd, London, 1991) pp. 549-568

Cottrell crack model for "unfavorable" {110} cracking in MgO



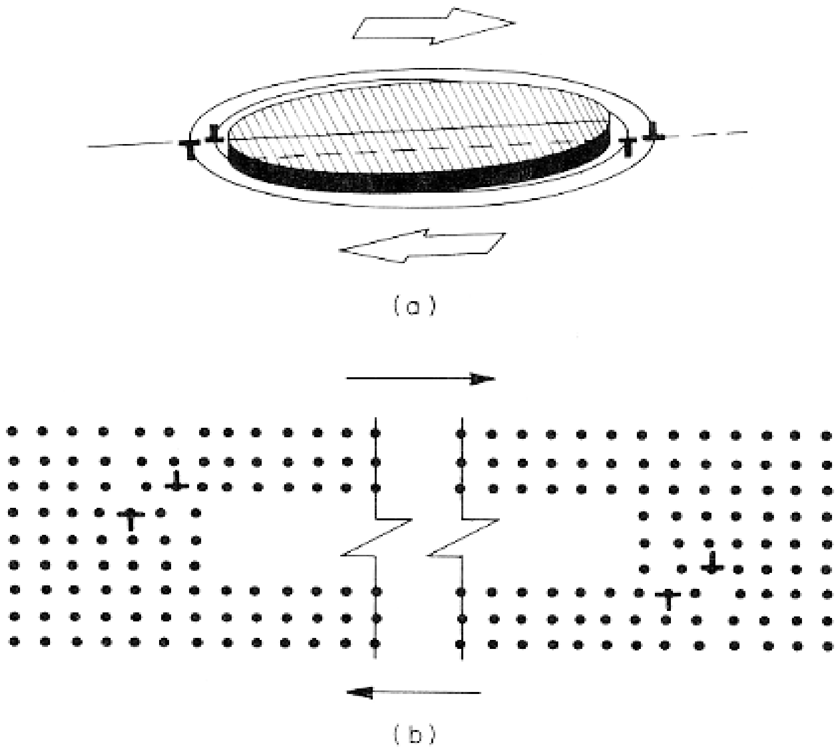
Top inclined crack: $(a/2)[10\bar{1}]_{(101)} + (a/2)[0\bar{1}1]_{(011)} = (a/2)[1\bar{1}0]_{(112)}$

B.L. Hammond and R.W. Armstrong, *Philos. Mag. Lett.*, **57**, 41-47 (1988); after, Y.T. Chou, F. Garofalo and R.W. Whitmore, *Acta Metall.*, **8**, 480-488(1960).

TOPICS 3

Limiting strength and brittleness of crystals

Theoretical Strength vs Smallest Crack Behaviour



Note by Wei Zhou: The paper assesses whether a crystal can break in a fully brittle manner or whether some plastic flow must accompany fracture. The criterion proposed is that, if the ratio of the largest tensile stress to the largest shear stress close to the tip of an equilibrium crack in the crystal is greater than the ratio of the ideal cleavage stress to the ideal shear stress, then a fully brittle fracture is possible. If the converse is the case, the crystal must always break with some plastic flow.

Brittleness Index: $\{\gamma/Gb\}^{1/2} \leq 0.29$

A. Kelly, W. R Tyson and A.H. Cottrell, “Ductile and Brittle Crystals”, *Philosophical Magazine*, **15**, 567 (1967); R.W. Armstrong, dislocation nucleation at a crack tip, *Mater. Sci. Eng.*, **1**, 251 (1966)

Crack growth with an associated plastic zone

Bilby, Cottrell and Swinden (1963):

$$(s/c) = [\sec(\pi\sigma_F/2\sigma_y)] - 1$$

in which s is length of plastic zone, c is crack half-length, σ_F is fracture stress and σ_y is yield stress.

Griffith-form; with $\sigma_y \rightarrow \sigma_{F0}$, crack-free fracture stress:

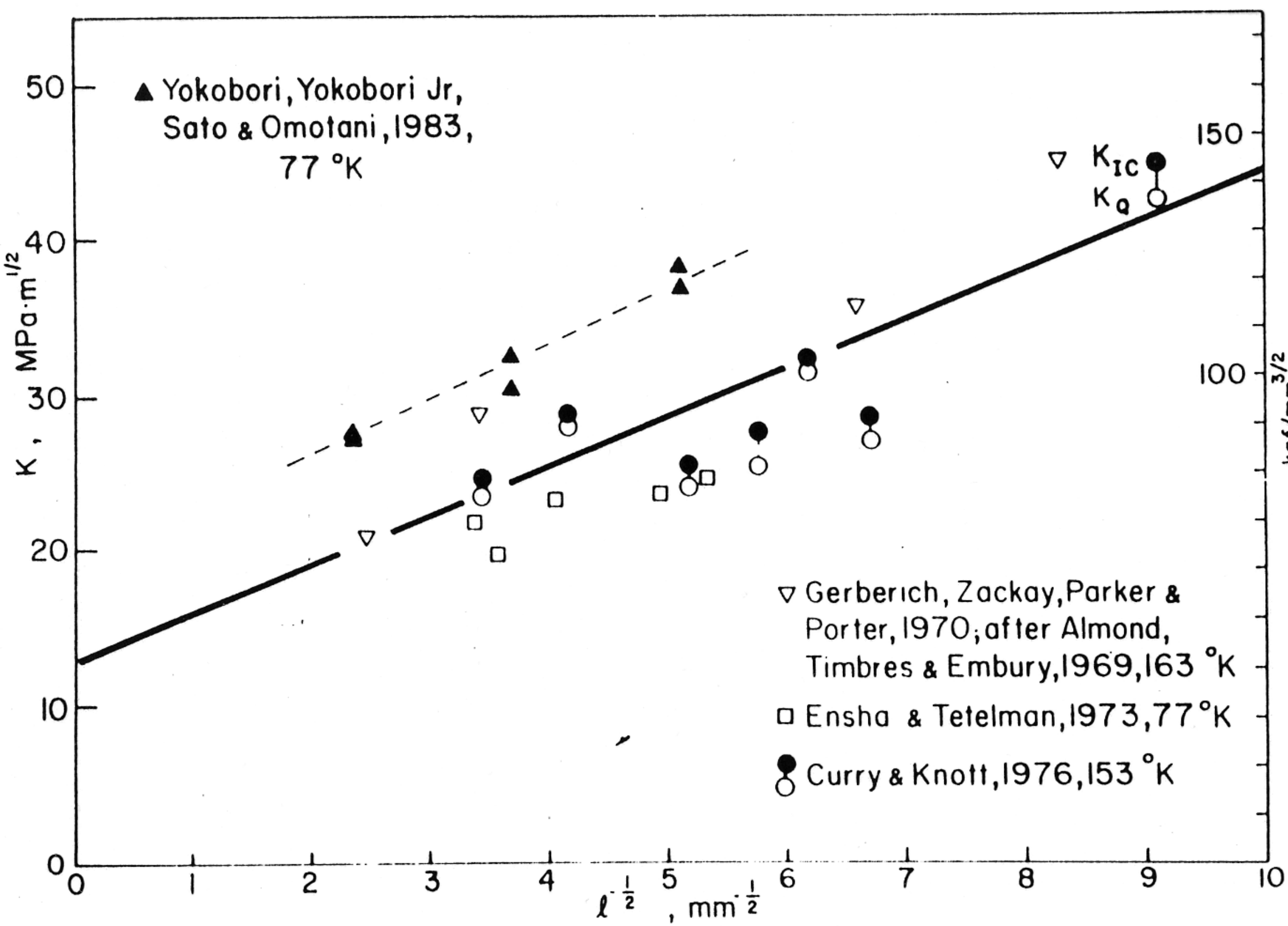
$$\sigma_F \approx A\sigma_{F0}\,[s/(c + s)]^{1/2}$$

Fracture mechanics stress intensity:

$$K_{Ic} = (8/3\pi)^{1/2}[\sigma_{0C} + k_C\ell^{-1/2}]s^{1/2}$$

G.R. Irwin, *7th Sagamore Ordnance Materials Research Conference* (Syracuse Univ. Res. Inst., N.Y., 1961) p. 63; R.W. Armstrong, *Eng. Fract. Mech*., **77**, 1348 (2010).

Hall-Petch relation for the Fracture Mechanics Stress Intensity



R.W. Armstrong, in: *Eng. Fract. Mech.*, **28**, [5-6], 529 (1987); *A Special Issue in Honor of Professor Takeo Yokobori on the Occasion of His 70th Birthday*, pp. 481-816.

Wei Zhou’s Note on Bilby, Cottrell and Swinden Model (1963):

The paper is entitled “**The Spread of Plastic Yield from a Notch**”

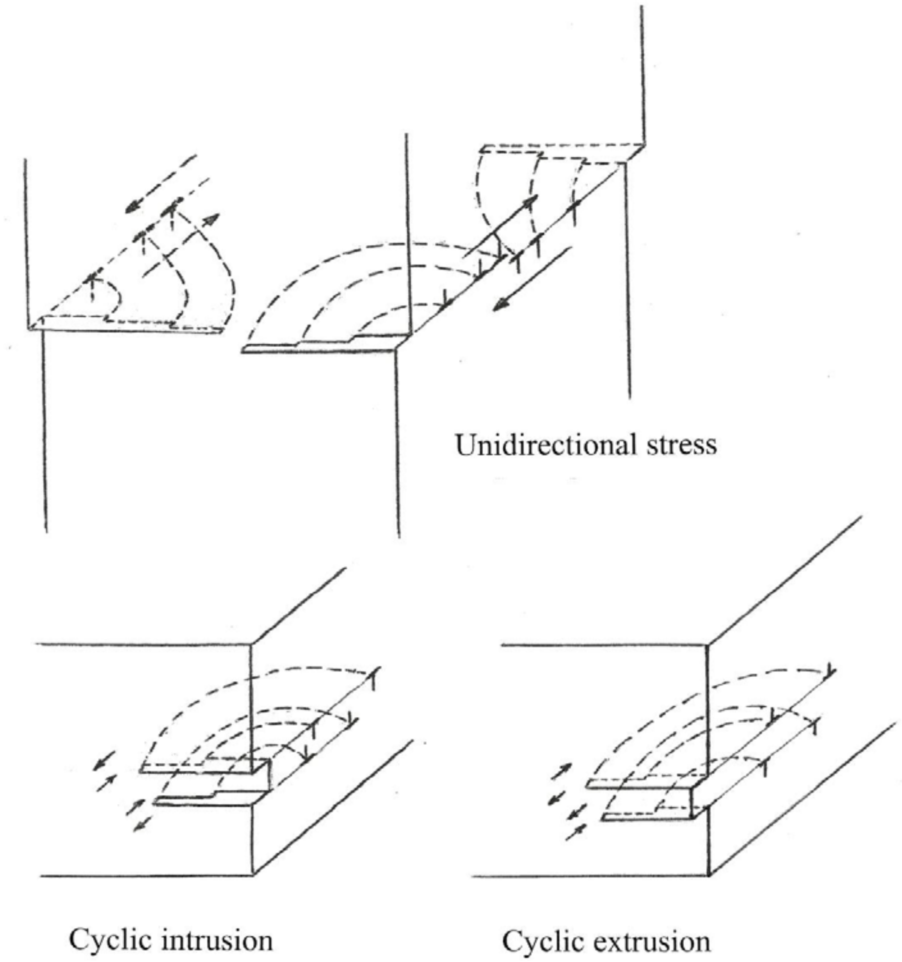
Two important contributions:

- The **concept of a “critical plastic stretch”,** before the notch breaks, which was also introduced by A. A. Wells as the “crack opening displacement”. COD has become an important design criterion.
- **An analytical solution** for estimating the lengths of plastic zones that need to span the cross section, for strong fractures.

TOPICS 4

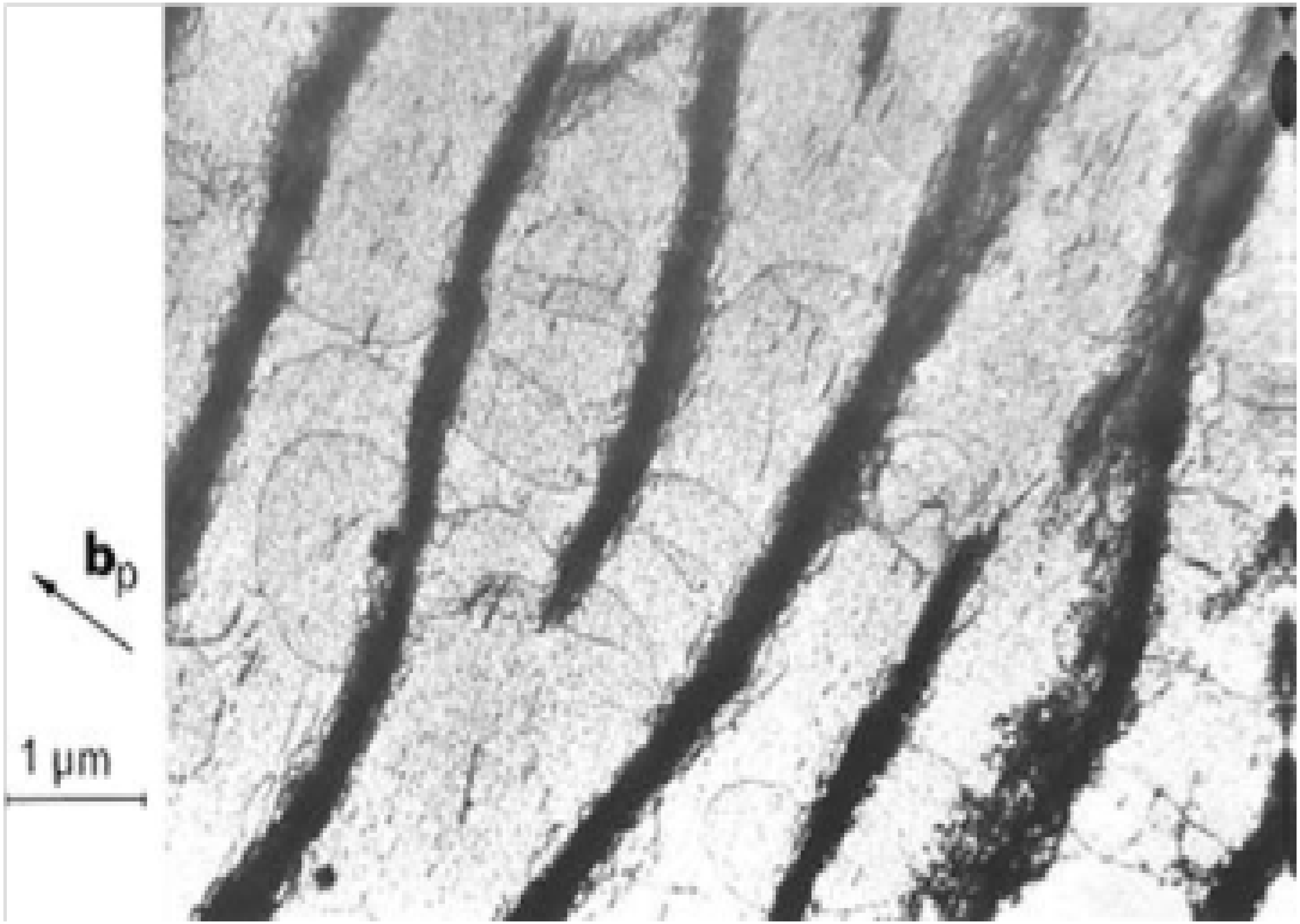
Slip band intrusions/extrusions in fatigue

[Cyclic Slip] to [Persistent Slip Bands, PSBs] to [Cracks]



A.H. Cottrell and D. Hull, Extrusion and Intrusion by cyclic slip in copper, Proc. Roy. Soc., Lond. A, 242, 211-213 (1957); S.D. Antolovich and R.W. Armstrong, Plastic strain localizations in metals; Origins and consequences, *Prog. Mater. Sci*., (2013) in print.

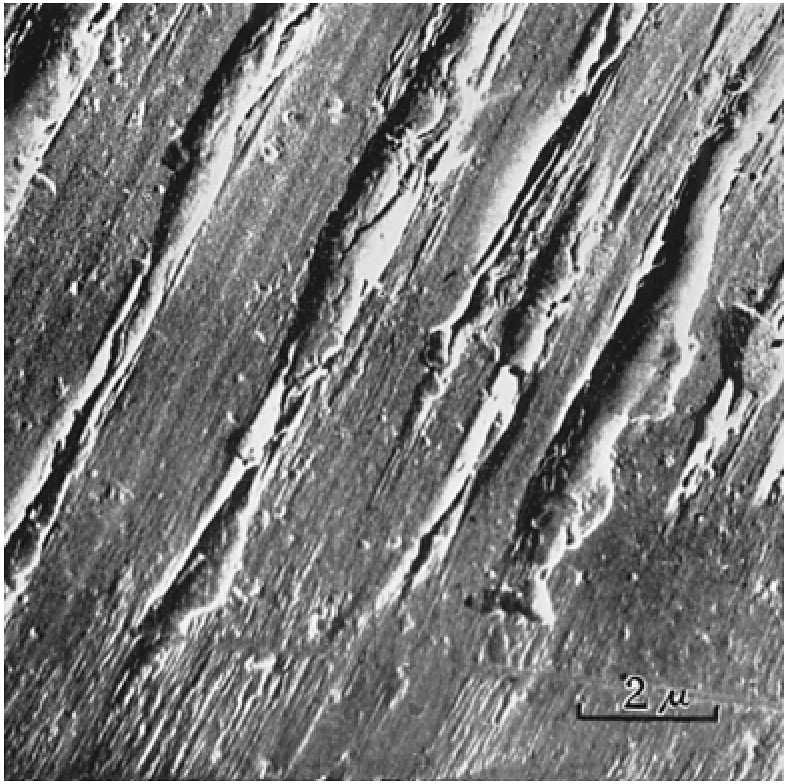
Dislocation Walls in Persistent Slip Bands

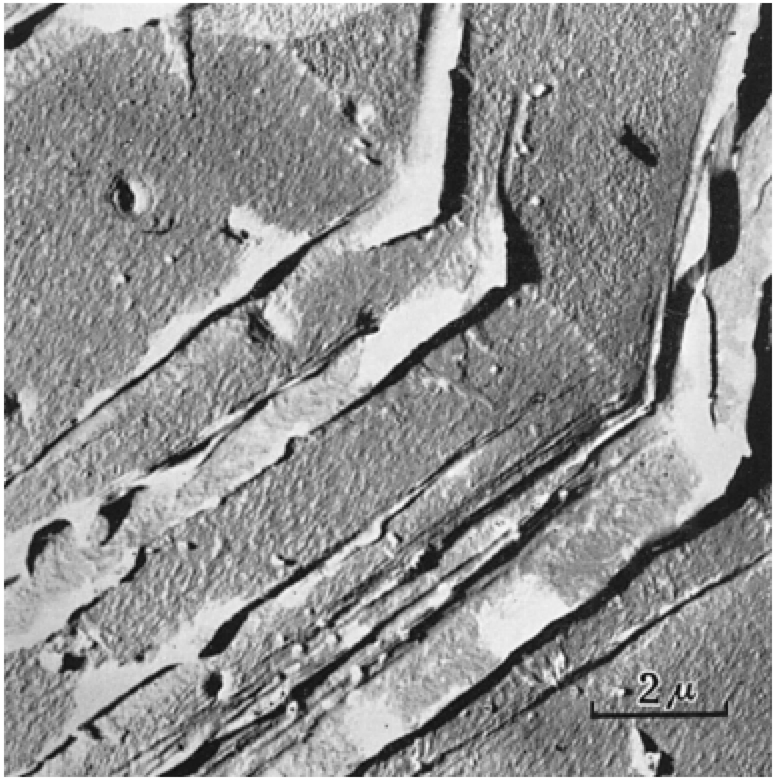


H. Mughrabi, F. Ackermann and K. Herz, "Persistent slip bands in fatigued fcc and bcc metals", in: *Fatigue Mechanisms*, ed. J.T. Fong (ASTM, Phila., PA, 1979), ASTM STP 675, pp. 69-105.

Note by Wei Zhou:

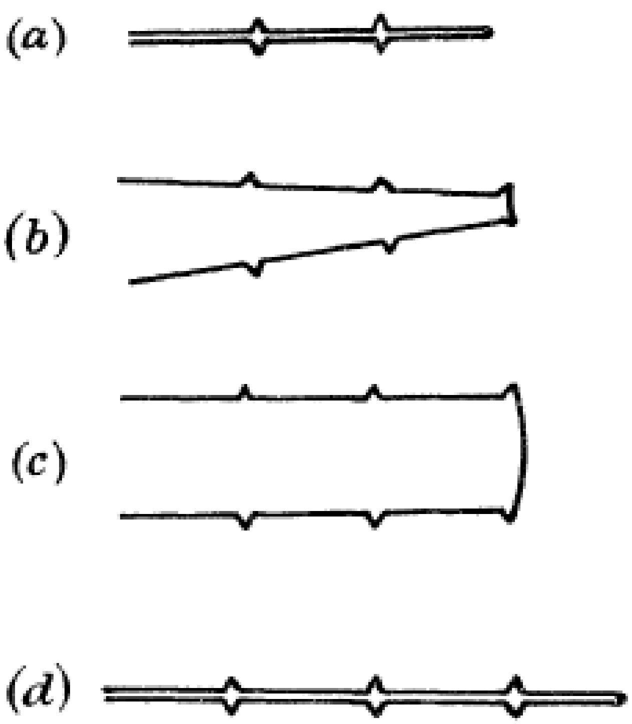
A.H. Cottrell and D. Hull's paper provides experimental evidence of extrusion (left) and intrusion (right) along the slip bands:

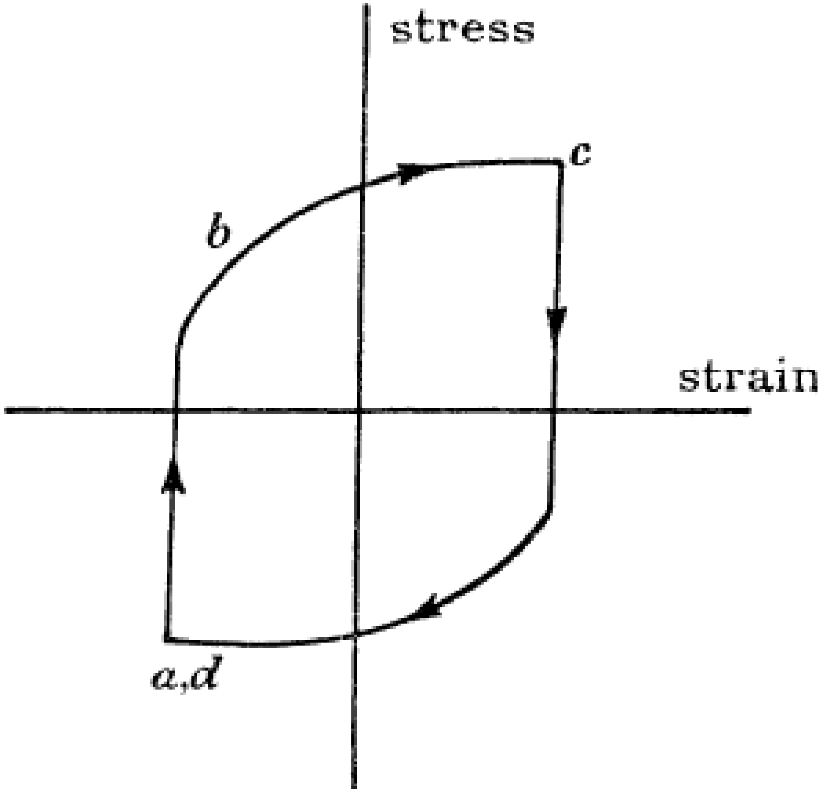




Note by Wei Zhou:

A.H. Cottrell and D. Hull's paper on fatigue was published in 1957. The fatigue model by C. Laird and G. C. Smith was published later in 1962:





SUMMARY

A number of Sir Alan Cottrell's leading contributions, with colleagues too, on the dislocation mechanics of fracturing have been briefly described:

(1) Conditions for a ductile-to-brittle transition in steel and related metals;

(2) Evaluation of the theoretical limiting strength and brittleness of crystals;

(3) Dislocation-modeled crack growth on a fracture mechanics description;

(4) Geometrical aspects of cyclic persistent slip band structures in fatigue.

SUMMARY

The topics provide cogent examples:

(1) of the importance of original insights provided by Cottrell in producing a better understanding of relevant issues; and,

(2) show a positive connection with further developments already made or continuing to be made on the same subjects.

SUMMARY

In this regard, a note is added from the interview of Sir Alan at ICF4 in 1977 in Waterloo, Canada, to the effect that more work needed to be done on multiple dislocation group dynamics, a topic that has been pursued in the interim time period and continues, as shown here, to be an important research activity.

The following slides are added by Wei Zhou to reflect his own personal view on Cottrell’s Dislocation Model in a historical framework

Why does Material Break?

Theoretical strength of solids?

$$\sigma_f \approx \frac{E}{10}$$

Actual strength?

- **Much lower!**

Why?

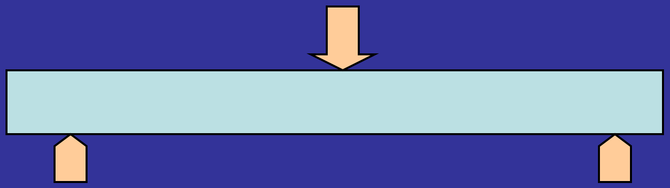
- **Due to Defects**

Questions:

- **How Defects are Formed**
- **How Defects Cause Fracture**

Mohist Propositions on Fracture (China, 2000 years ago)

- **(Suppose a weight) to be supported (by a beam) which does not break. The reason is given under "bearing".**



- **(It is upon) evenness, or continuity, that breaking or not breaking depends. The reason is given under** "evenness, or continuity".

Mohist might mean:

Defects are responsible for fracture, but it is a philosophical approach.

Controlled Experiments in the West

Leonardo DaVinci (1452-1519):
short iron wires were noticeably stronger than longer ones.

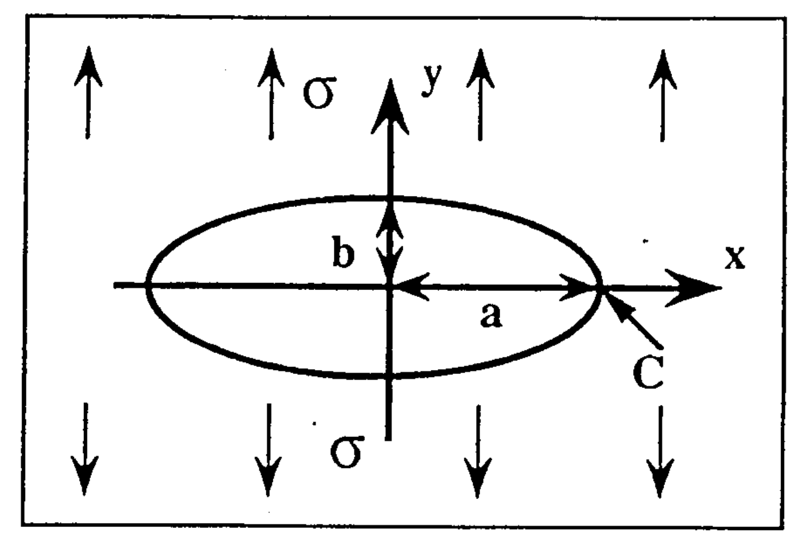
Good observation, but no fracture modelling

Why does Defect Reduce Strength?

Defects → Stress Concentration

The Inglis analysis (1913):

$$\sigma_{max} = \sigma(1 + 2\frac{a}{b})$$



$$\rho = \frac{b^2}{a}$$

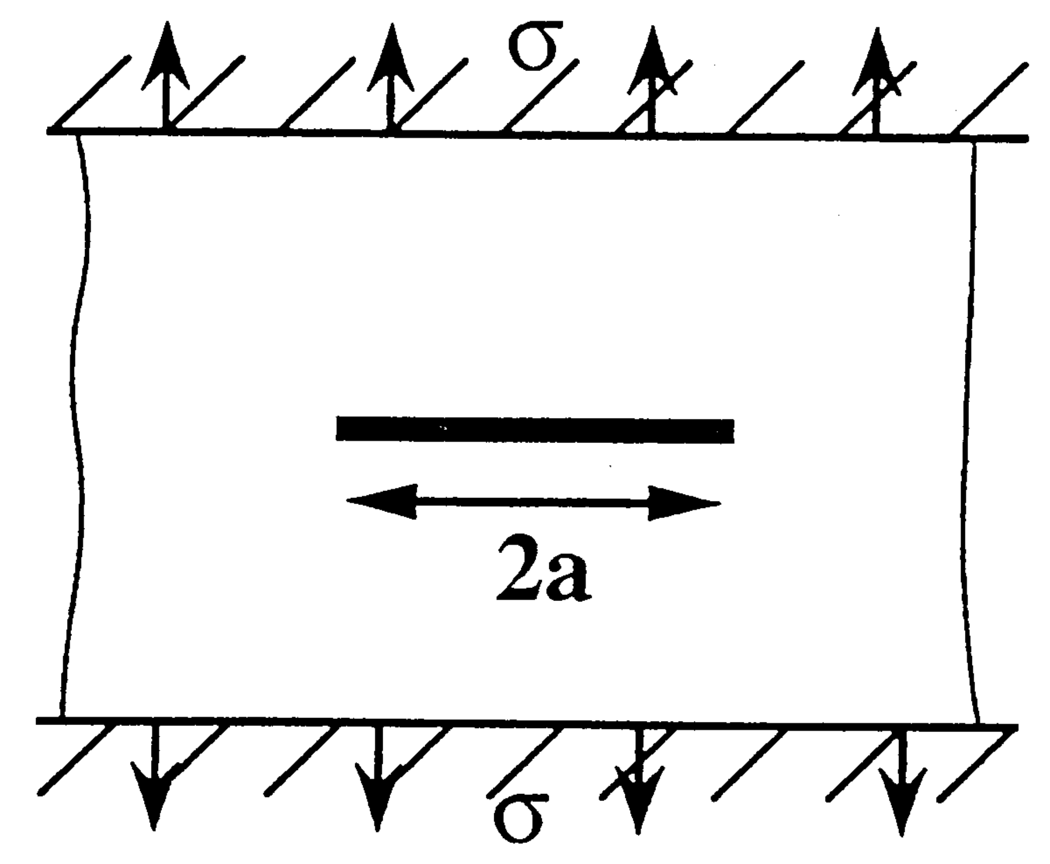
$$\sigma_{max} = \sigma(1 + 2\sqrt{\frac{a}{\rho}})$$

Any problems?

- **The stress concentration depends on the shape rather the size of defect,** ***contradicting the fact that large defects tend to propagate more easily than small ones.***
- ρ? It is difficult to explain the ***physical significance of the radius of curvature*** ρ at the tip of a real crack.

The Griffith Equation (1920's)

Energy-Balance Argument



$$\frac{dU}{da} = 0$$

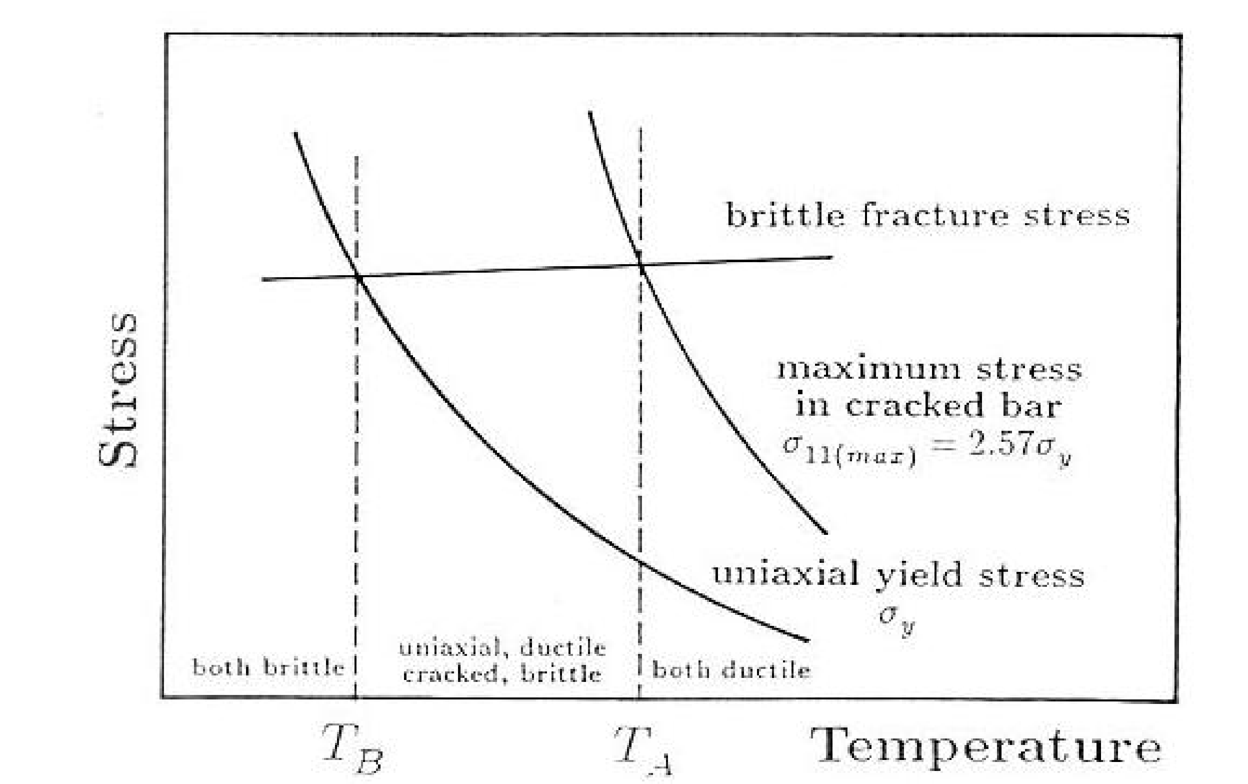
The *critical* stress to propagate the crack is:

$$\sigma_f = \left(\frac{2E'\gamma_p}{\pi a} \right)^{\frac{1}{2}}$$

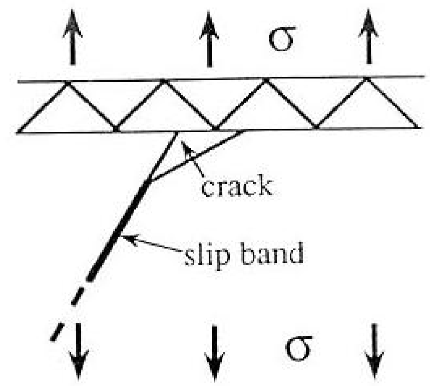
$$E' = E$$ for plane stress condition

$$E' = \frac{E}{1-\nu^2}$$ for plane strain condition.

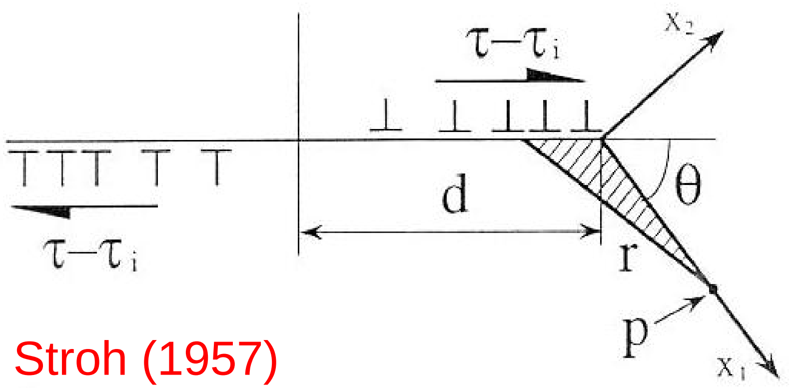
The Tensile Fracture Criterion



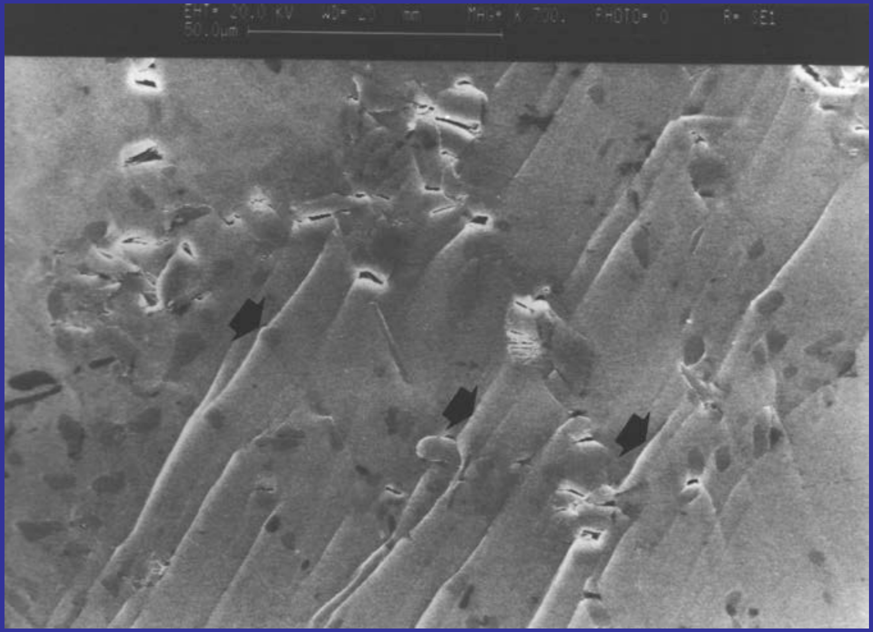
Dislocation Models



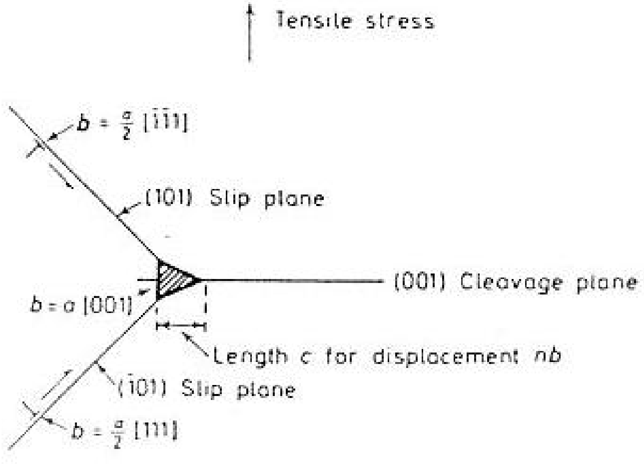
Zener (1948)



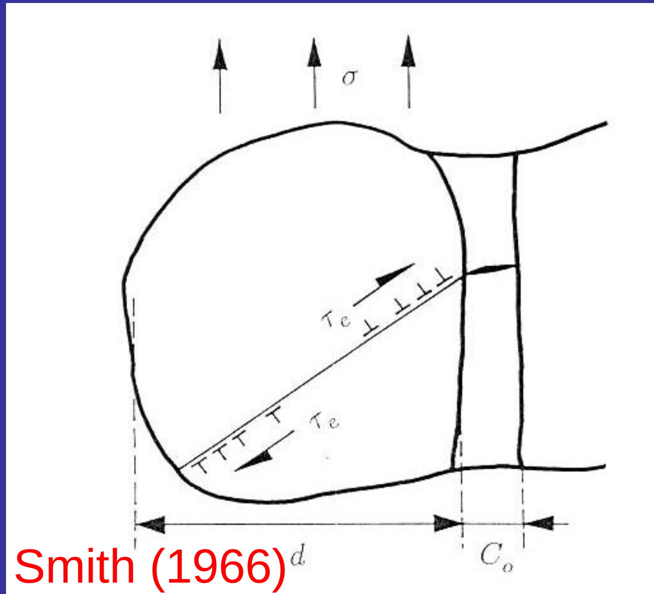
Stroh (1957)

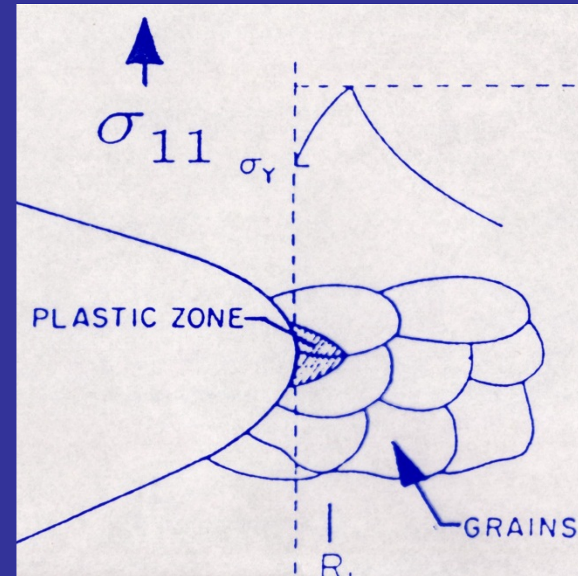


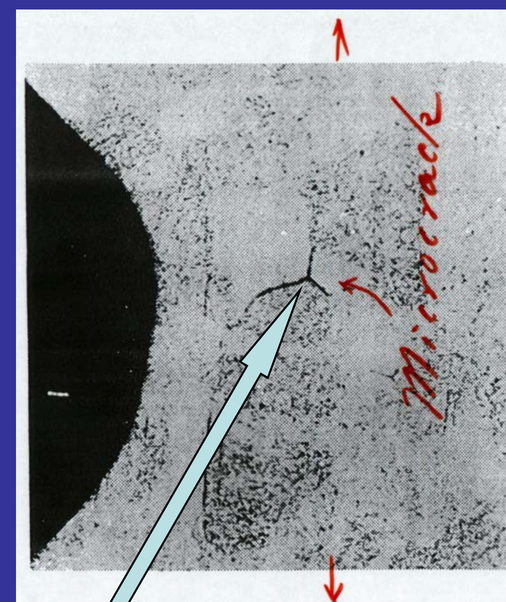
Experiment by Wei Zhou



Cottrell (1958)

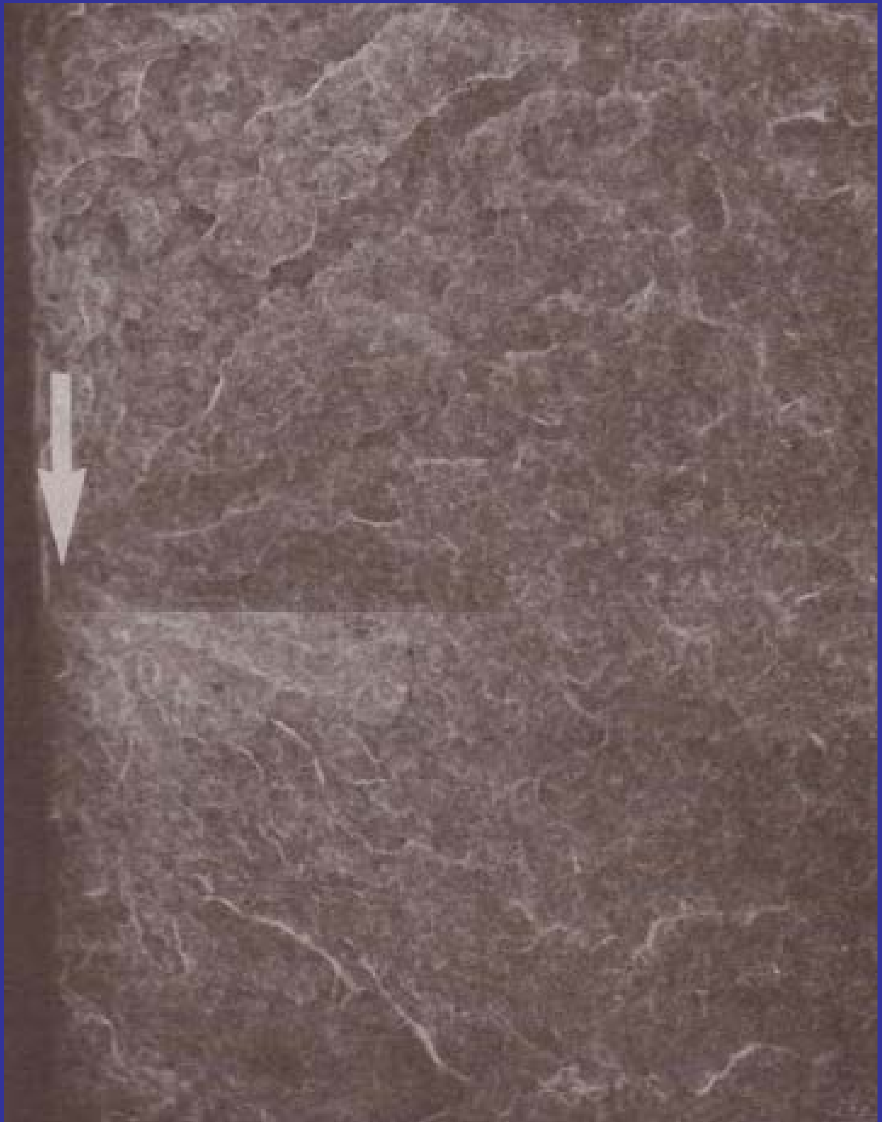






Fracture Initiates ahead of notch root (J. F. Knott)

Example of Fracture Initiation ahead of Notch Root



Wei Zhou and J. F. Knott, HY80 Steel

Comments on Dislocation Models

- Zener's and Stroh's models predict that crack formation is the most difficult stage in fracture, so the fracture is initiation-controlled.
- This is at variance with the experimental results for most materials.

Comments on Dislocation Models

- Cottrells' model predicts a tensile stress controlled cleavage. It also explains the effect of grain size and yielding parameters on fracture.
- Cottrell's model can explain cleavage fracture in single crystals, which does not involve grain boundaries as barriers to dislocation pile-ups.