# Wave Propagation Through Soft Tissue Matter

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# Background and Motivation

*Brain injury mechanisms associated with direct or indirect (sudden acceleration/deceleration) impacts involve relative rigid body motions of the brain with respect to the skull.*

- For low speed impacts (relatively large time scale) , brain injury mechanisms are governed by rigid body displacements within the skull.

- For high speed impacts (small time scale), deformation of the brain has been proposed as a primary mechanism leading to injury (Hong *et al*., 2007).

- Shock waves reaching an object result in loading transients involving much shorter characteristic time scales, probably leading to high strain rates as compared to those during impacts conditions ( Nyein *et al*., 2008).

*Impact and blast waves promote inherently different brain injury mechanisms*

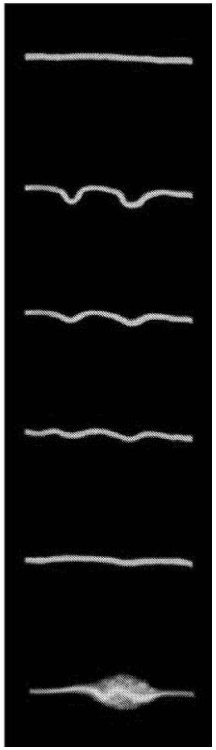
# Brain Tissue and Axon Behavior

- Strain rate levels ranging from 15 sec$^{-1}$ to 21 sec$^{-1}$ have been associated with the occurrence of axonal injury. Axonal injury due to bending distortions were experimentally observed for large strain rates; provide evidence for viscoelastic nature of the axonal behavior: Chetta *et al.* (2010) and Tang-Schomer *et al.* (2011).

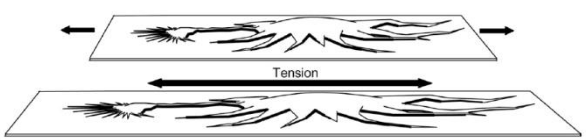
- Experiments with swine and rat brain tissue in compression, tension and shear: Mendis *et al.* (1995), Miller and Chinzei (2002), Brands *et al.* (2004) and Pervin and Chen (2007)

- Anatomically detailed finite elements model studies: Brands (2002) Zhang *et al.* (2004) and El Sayed *et al.* (2008).

- Numerical simulations of bTBI: Moore *et al.*(2009), Taylor and Ford (2009) and Nyein *et al.*(2010); Experimental bTBI: Alley *et al.*(2011).

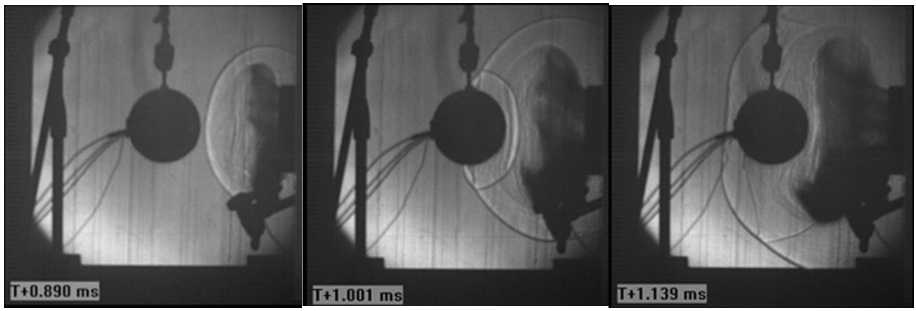


**Axonal injury (Smith and Meaney, 2000)**




**Tensile experiments with axons (Chetta *et al.* 2010)**


**ANSYS Simulation, CECD**




**Experimental Blast (Alley *et al.* 2011)**

# Current Work: Overall Aim and Specific Objectives

**Overall Aim**: Develop a fundamental understanding of the effect of blast waves on the human skull-brain system; in particular, with regard to nonlinear wave propagation in brain tissue and activation of unique brain injury mechanisms

**Specific Objectives**:

•Develop reduced-order models to study wave propagation along brain fibers to complement complete full model studies being conducted at NSWC, Indian Head. Understand the **influence of nonlinear material properties on wave propagation** in soft tissue.

• Obtain **fundamental insights into the propagation of mechanical (stress/strain) waves along brain fibers**. Link frequency content and amplitude of external loading with the occurrence of injury at different length scales (brain fibers, axons, and so on). Use this analysis to determine appropriate loading amplitudes and frequencies for experiments that attempt to replicate blast loading conditions at UMD and UMB.

• Integrate medical imaging (MRI and CT scans), nonlinear material constitutive models, and finite element analysis.

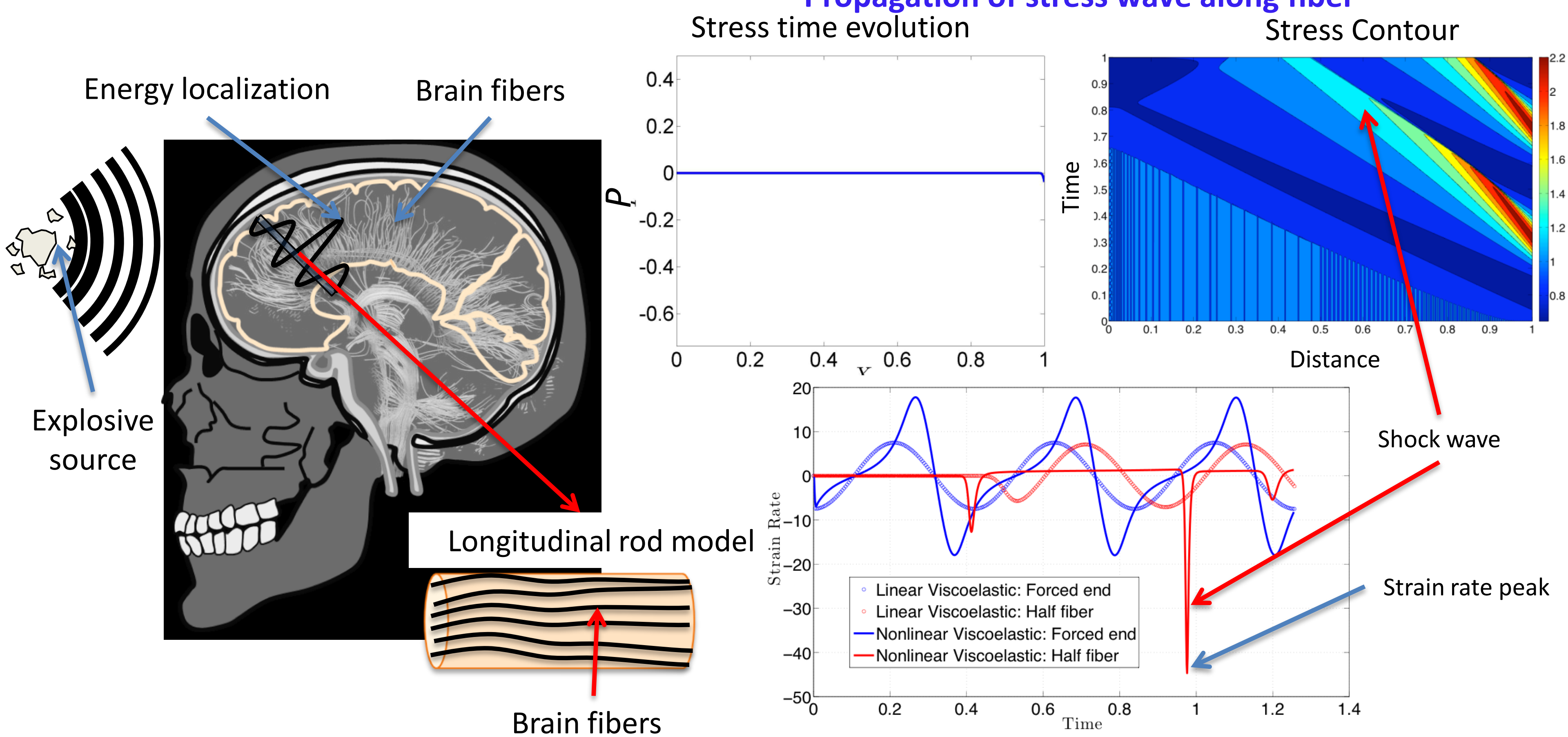
# Wave Propagation in Brain Fibers

✓Which **frequency components** and **amplitude levels** of external loading favor energy localization?

✓ Can we link the predictions on wave propagation along brain fibers with the occurrence of injury?

✓ Can the model predictions guide the choice of **appropriate loadings** (amplitude and frequency) in **brain tissue experiments** so that a particular phenomenon is observed?

**Propagation of stress wave along fiber**



# Wave Propagation in Brain Fibers

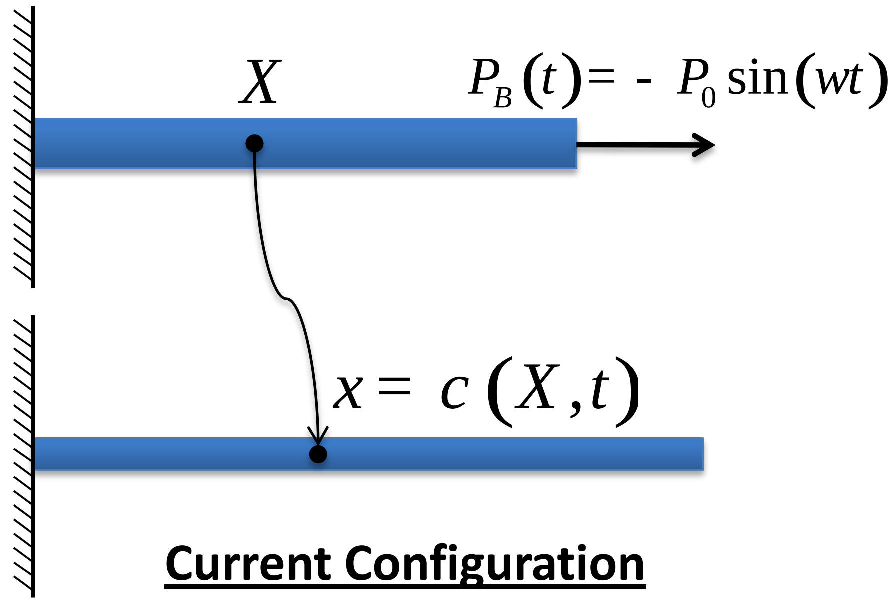
## Governing Equation

$$\rho_0 \frac{\partial^2 \chi}{\partial t^2} = \frac{\partial P}{\partial X}$$

$P$ : uniaxial first Piola-Kirchhoff Stress

$\rho_0$ : original density

## Reference Configuration




•Uniaxial deformation gradient

$$F = \frac{\partial \chi}{\partial X} = \lambda \quad \text{: Stretch ratio} \quad 0 < \lambda$$

•Hyper-elastic Long term behavior

$$P = \frac{\partial \Psi(\lambda)}{\partial \lambda}$$

$\Psi$ : Material Strain Energy Function

•Nonlinear Viscoelastic Constitutive Equation

$$\dot{\lambda} = \frac{d\lambda}{dt} = -k \left[ \frac{\partial^2 \Psi}{\partial \lambda^2} \right]^{-2} \left[ \frac{\partial \Psi}{\partial \lambda} - P \right] \qquad (*)$$

$k > 0$ : Relaxation Modulus

(*) Haslach, W.H.. Thermodynamically consistent, maximum dissipation, time-dependent models for non-equilibrium. *International Journal of Solids and Structures* 46 (2009) 3964-3979

# Wave Propagation in Brain Fibers

**Governing equations**

$$
\begin{cases}
\rho_0 \dfrac{\partial^2 \chi}{\partial t^2} = \dfrac{\partial P}{\partial X} \\
\dot{\lambda} = -k\left[g(\lambda)\right]^{-2}\left[f(\lambda) - P\right] \\
\dfrac{\partial \chi}{\partial X} = \lambda
\end{cases}
$$

**Boundary Conditions**

$$\chi(0,t) = 0$$

$$P(L,t) = P_B(t)$$

**Initial Conditions**

$$\chi(X,0) = 0$$

$$\frac{\partial \chi(X,0)}{\partial t} = 0$$

$$f(\lambda) = \frac{\partial \Psi}{\partial \lambda}$$

$$g(\lambda) = \frac{\partial^2 \Psi}{\partial \lambda^2}$$

$\Psi$ : Material strain energy function

e.g., Incompressible Mooney-Rivlin Hyper-elastic model

$$\Psi(\lambda) = c_1\left(\lambda^2 + 2\lambda^{-1} - 3\right) + c_2\left(\lambda^{-2} + 2\lambda - 3\right)$$

# Wave Propagation in Brain Fibers: Linear Model Predictions

- Strain Energy Function $\Psi(\lambda) = \frac{1}{2}E(\lambda - 1)^2$

$$\begin{cases} \rho_0 \dfrac{\partial^2 \chi}{\partial t^2} = \dfrac{\partial P}{\partial X} \\ P(\lambda, \dot{\lambda}) = E(\lambda - 1) + \dfrac{E^2}{k}\dot{\lambda} \end{cases}$$

$$\boxed{\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial x^2} + a\,\frac{\partial^3 v}{\partial t\,\partial x^2}}$$

Nondimensionalization of variables

$$v = \frac{\chi - X}{L} \qquad x = \frac{X}{L} \qquad t = \frac{t}{L/c_0}$$

$v$ : Nondimensional displacement

$t$ : Nondimensional time

$x$ : Nondimensional space

$c_0 = \sqrt{\dfrac{E}{\rho_0}}$ : Wave speed

$$a = \frac{Ec_0}{kL} \approx \left[\frac{\text{Dissipation time scale}}{\text{Wave propagation time scale}}\right]$$

# Wave Propagation in Brain Fibers: Linear Model Predictions

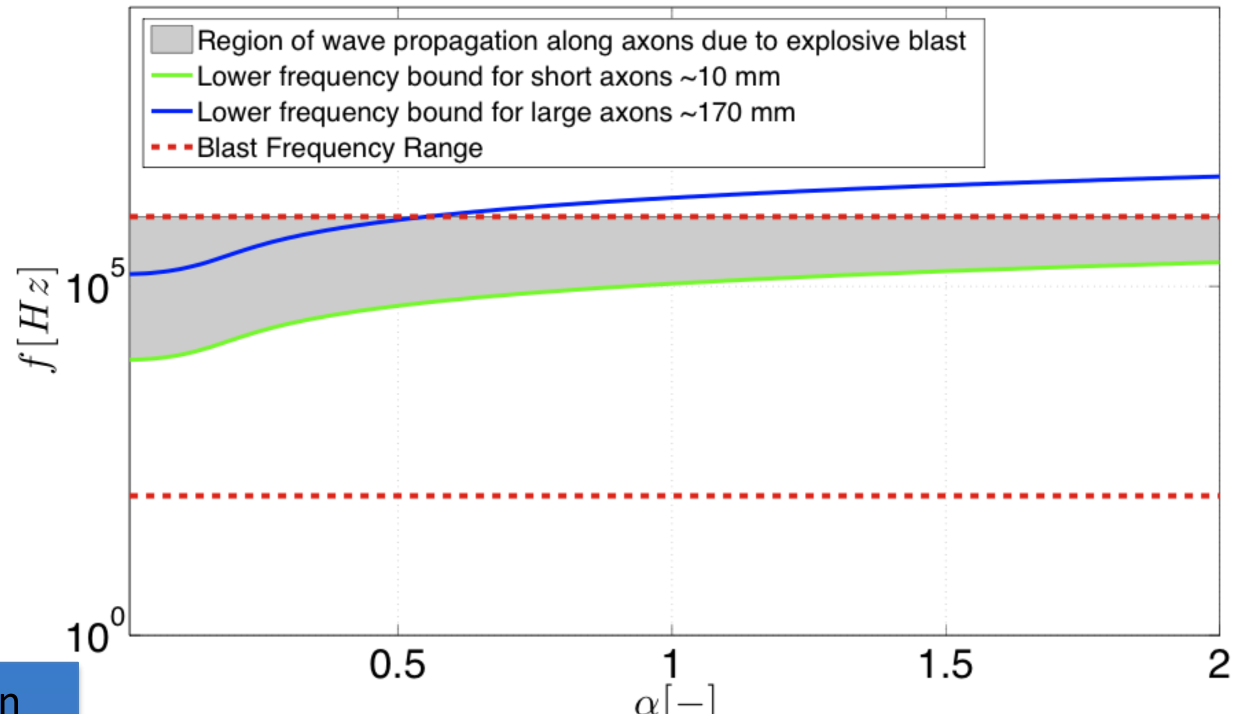
**•Exponential Solution** $$v(x,t) = e^{-A(\bar{\omega})\sin(1/2\phi)x}\, e^{iA(\bar{\omega})\cos(1/2\phi)\left[x + c(\bar{\omega})t\right]}$$

$\bar{\omega} = \omega L / c_0$ : Nondimensional frequency

$$A(\bar{\omega}) = \frac{\bar{\omega}}{\left(1 + \bar{\omega}^2\alpha^2\right)^{\frac{1}{4}}}$$

$$\phi(\bar{\omega}) = -\tan^{-1}\left[\alpha\bar{\omega}\right]$$

$$c(\bar{\omega}) = \frac{\sqrt{2}\left(1 + \alpha^2\bar{\omega}^2\right)^{\frac{3}{4}}}{\left(2 + \bar{\omega}^2\alpha^2\right)^{\frac{1}{2}}}$$



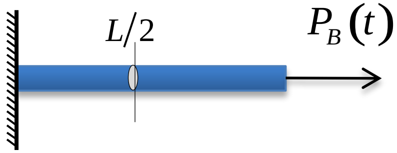

Lower Frequency Bound equation

$$\lambda_{\max} = 1 = \frac{2\pi c(\bar{\omega})}{\bar{\omega}} \Rightarrow \bar{f}(\alpha) = \frac{\bar{\omega}_{\mathrm{LB}}(\alpha)}{2\pi}$$

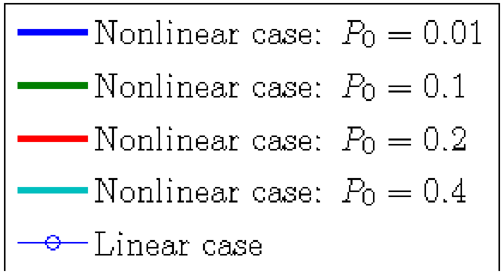
| Axons | | Unknown property |
|---|---|---|
| $L = 10\ \text{mm} - 170\ \text{mm}$ | $c_0 \approx 1500\ \text{m/sec}$ | $k = ? \rightarrow \alpha = ?$ |

# Wave Propagation in Brain Fibers

Effect of forcing frequency and forcing amplitude on the transient (wave propagation) response

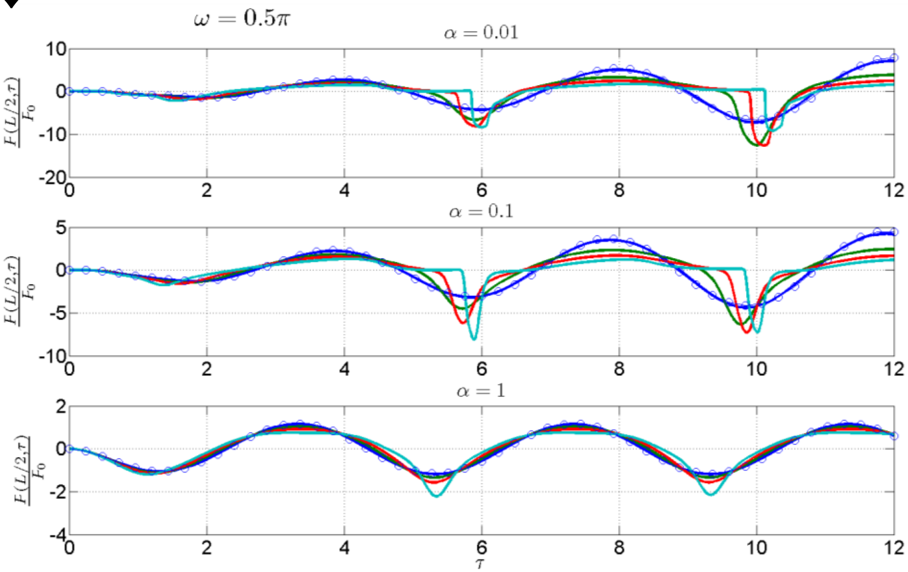



$P_B(t) = -P_0 \sin(wt)$

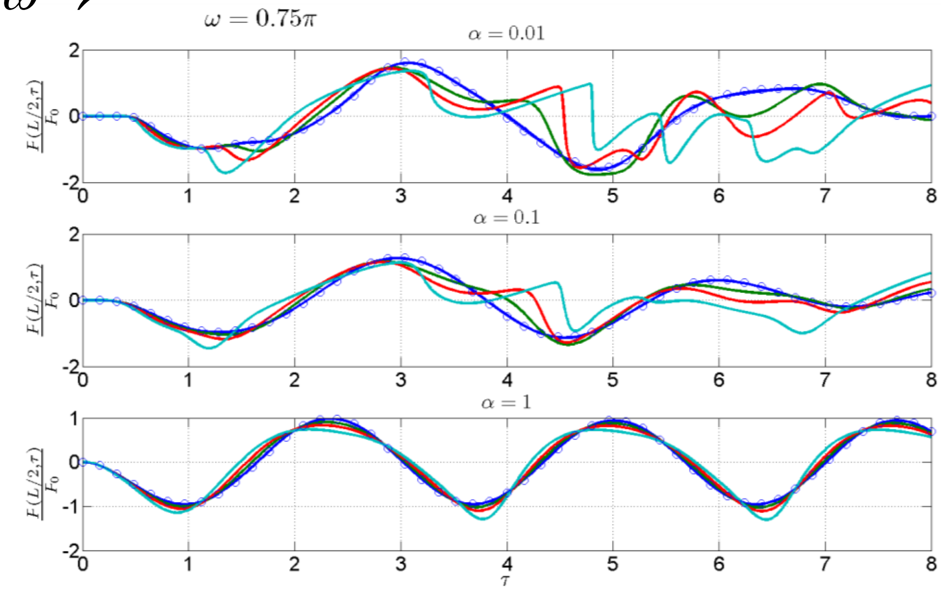



## Stress time histories at the half section of the fiber

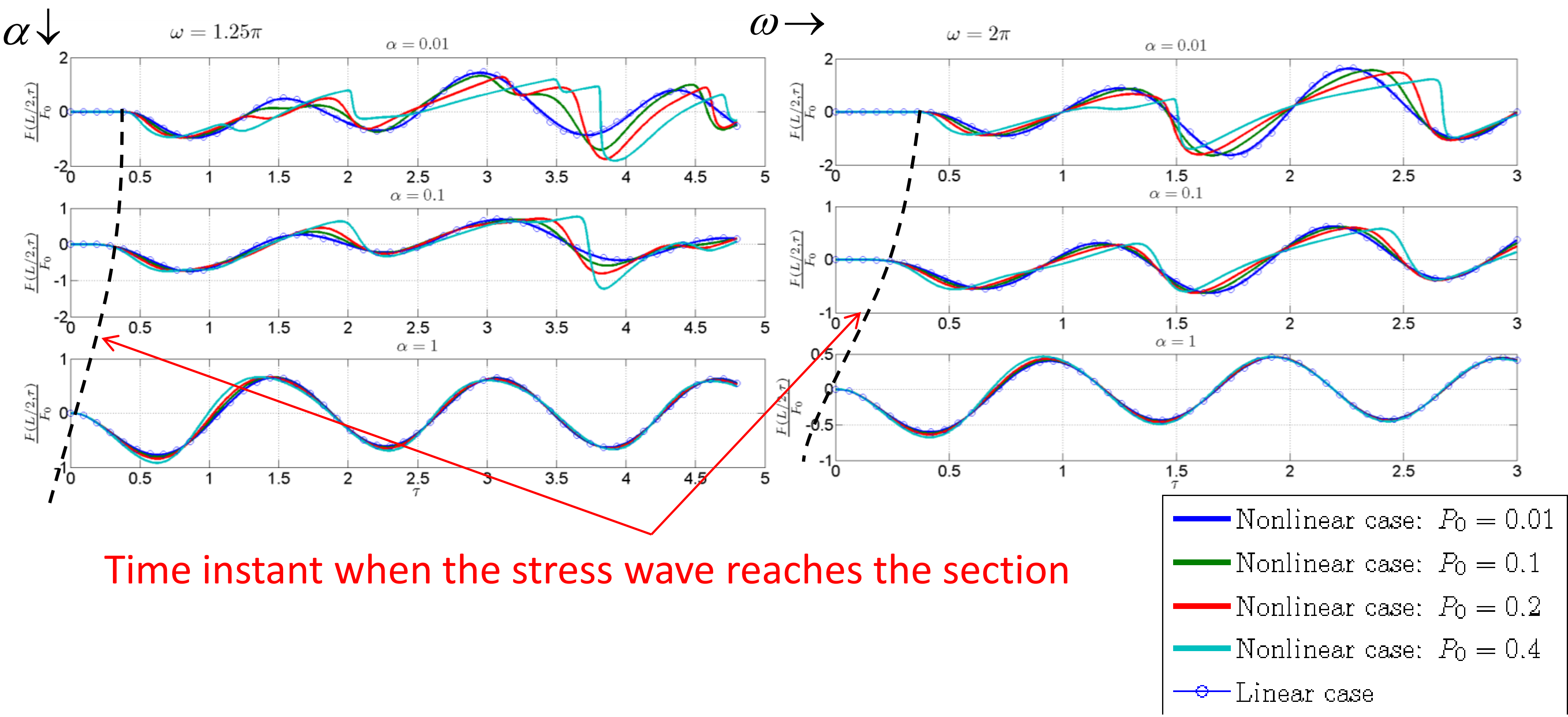
$\alpha \downarrow$




$\omega \rightarrow$



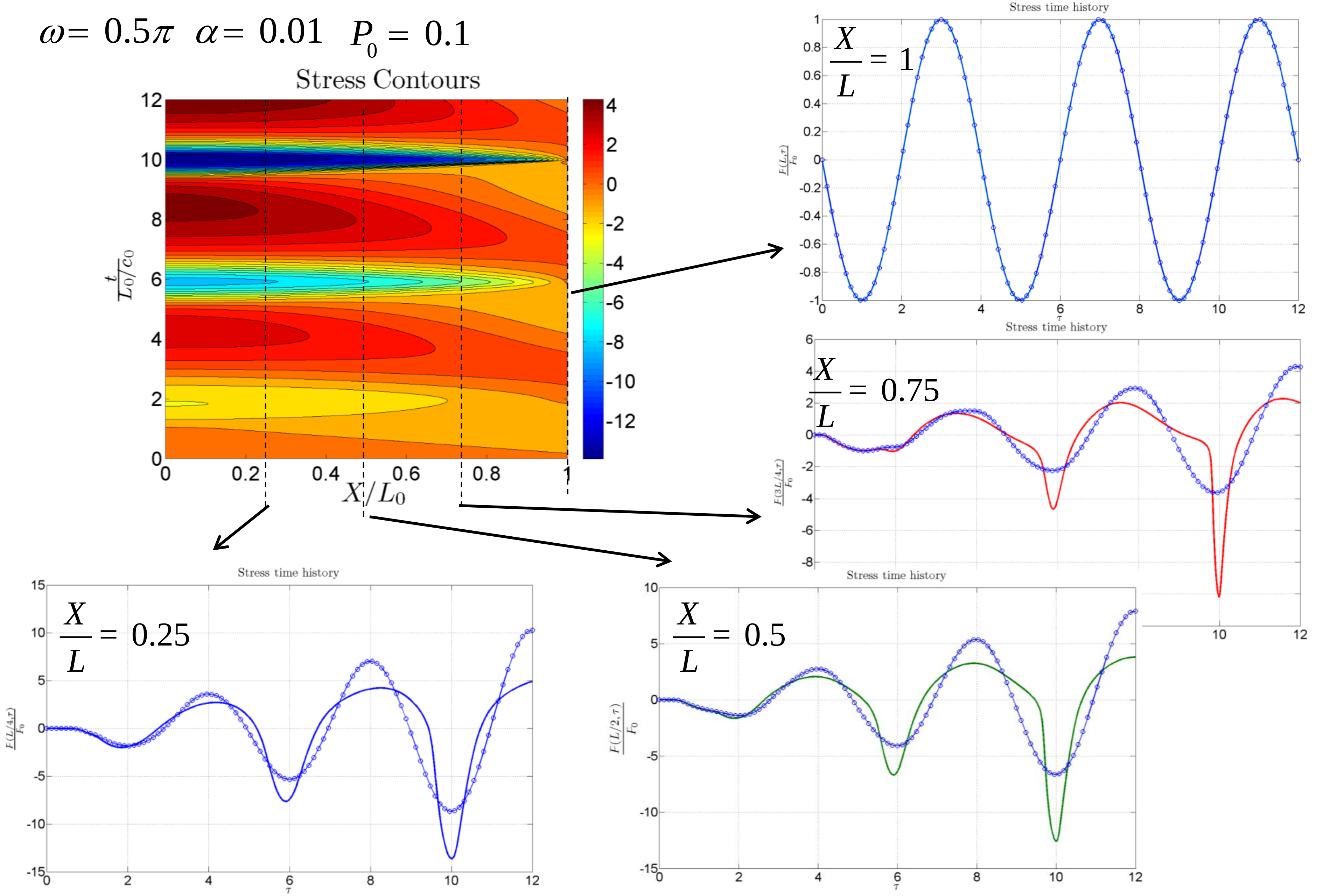
# Wave Propagation in Brain Fibers




- Increasing dissipation increases the speed of propagation of the system
- For $\alpha = 1$, transients are quickly dissipated as the frequency increases. For this case, linear and non-linear solutions become similar for all the amplitude load values analyzed.
- For large value of the amplitude load, there are discontinuous jumps as a consequence of both steepening of the waves (leading to shock formation) and interaction between incoming and reflected waves at the boundaries.

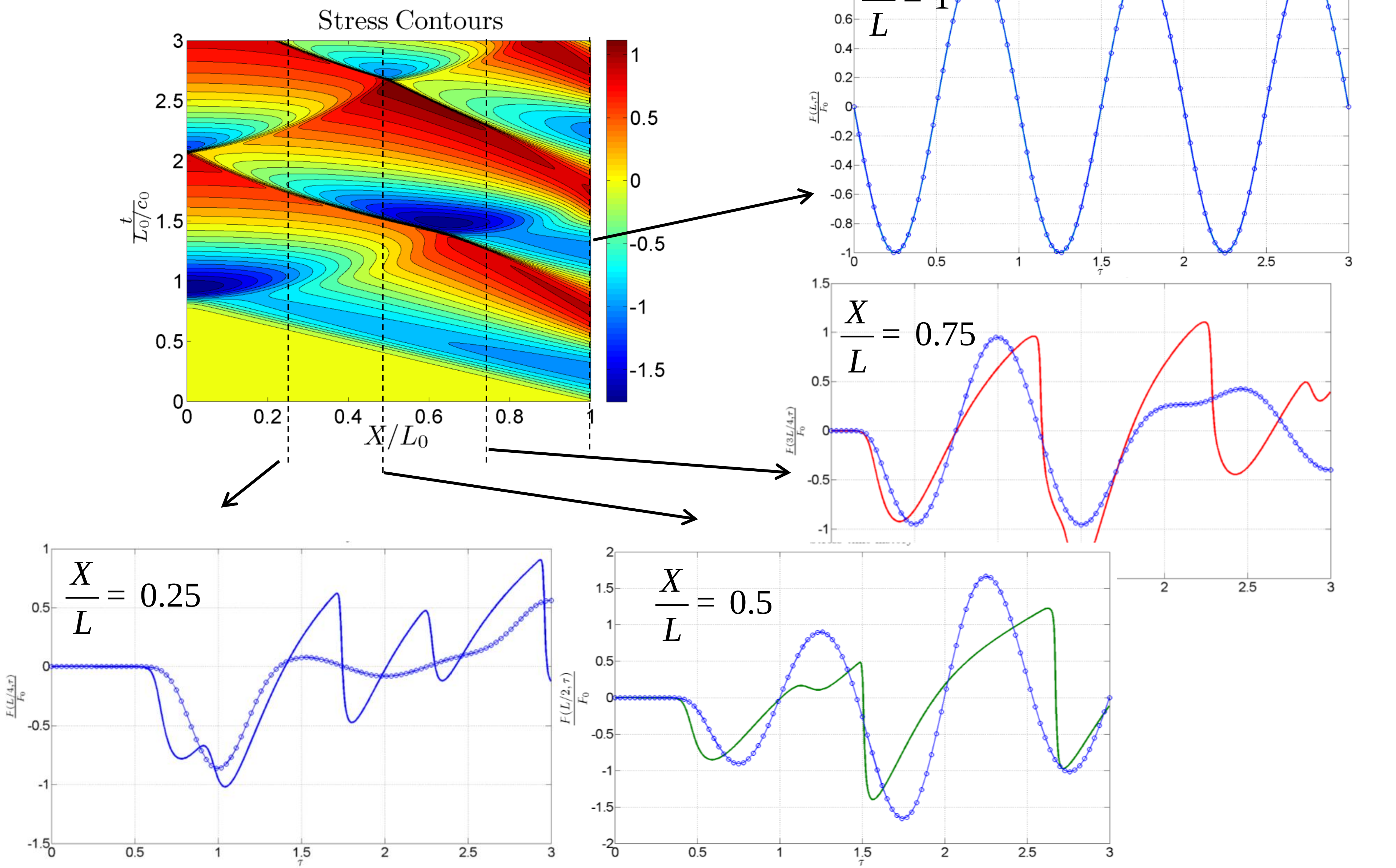
# Wave Propagation in Brain Fibers

$\omega = 0.5\pi \quad \alpha = 0.01 \quad P_0 = 0.1$



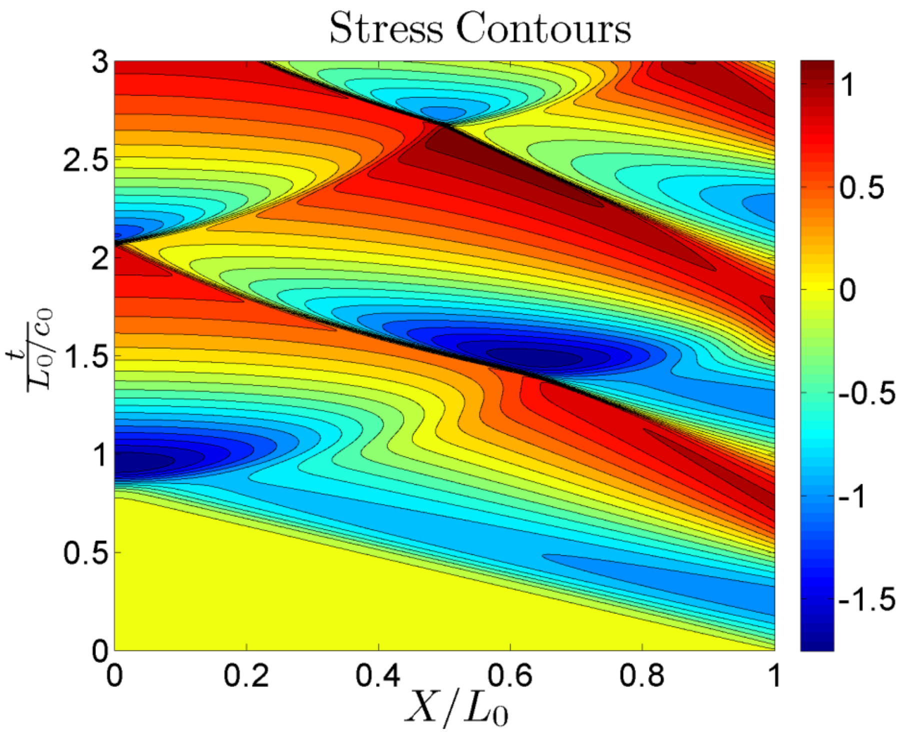
# Wave Propagation in Brain Fibers

$\omega = 2\pi \quad \alpha = 0.01 \quad P_0 = 0.4$

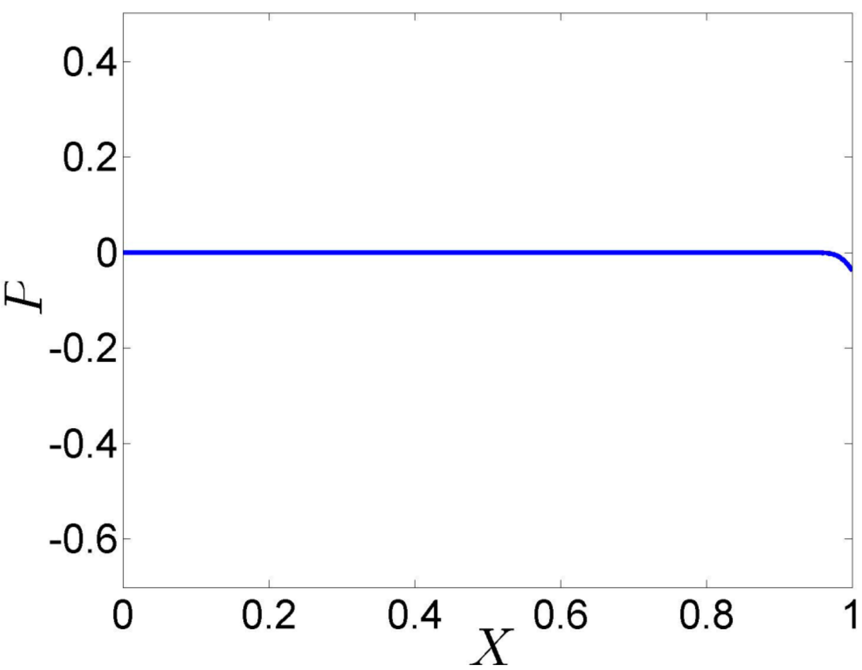


# Wave Propagation in Brain Fibers

$\omega = 0.5\pi \quad \alpha = 0.01 \quad P_0 = 0.1$



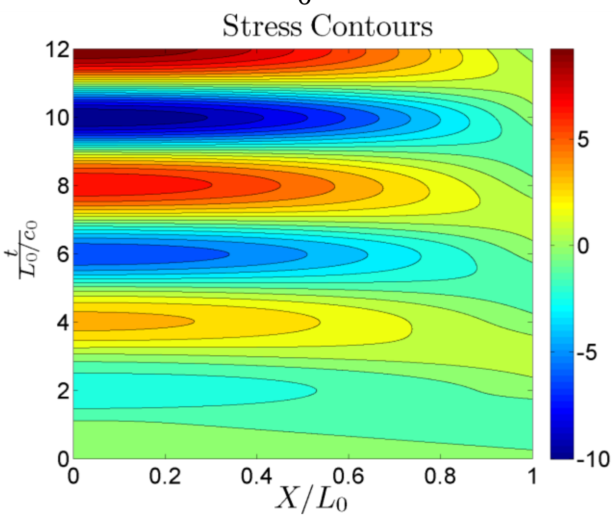

Stress as a function of position

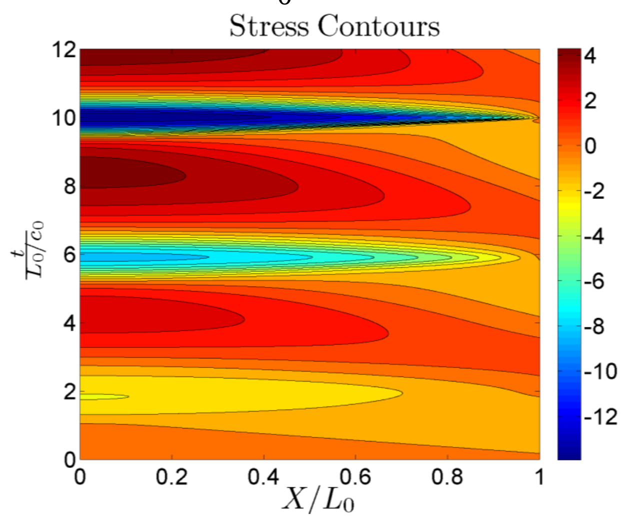


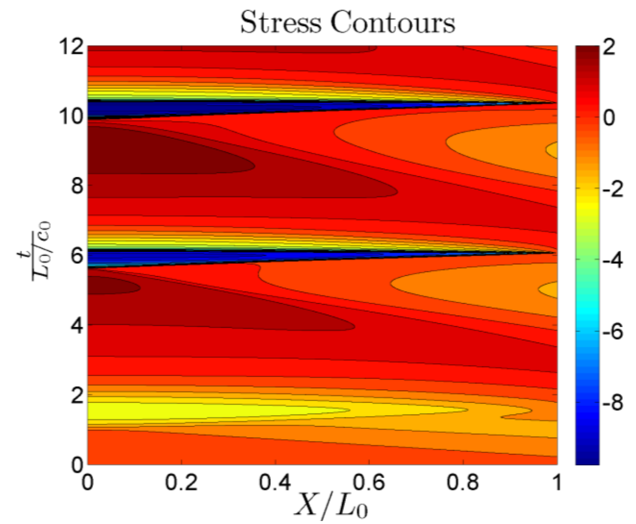
# Wave Propagation in Brain Fibers

$\omega = 0.5\pi$

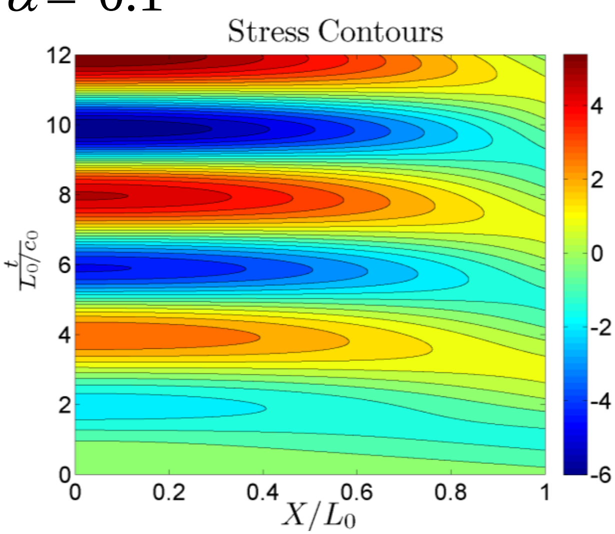
$\alpha = 0.01$ $\quad P_0 = 0.01$ $\quad P_0 = 0.1$ $\quad P_0 = 0.4$

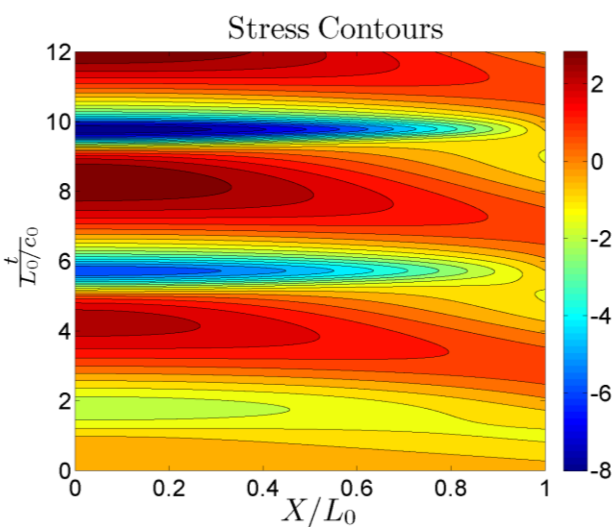


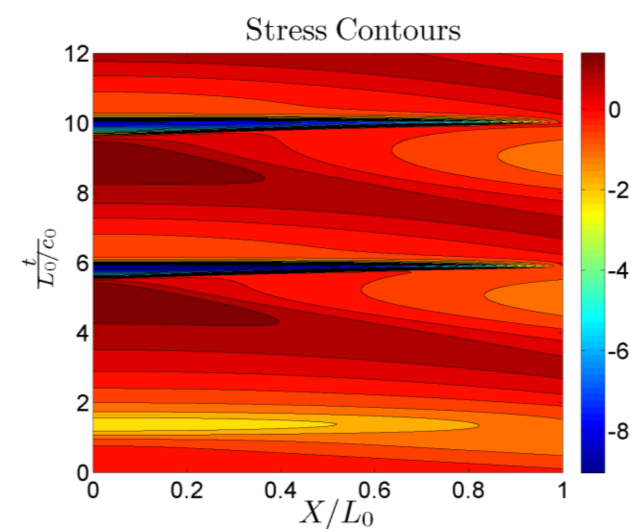



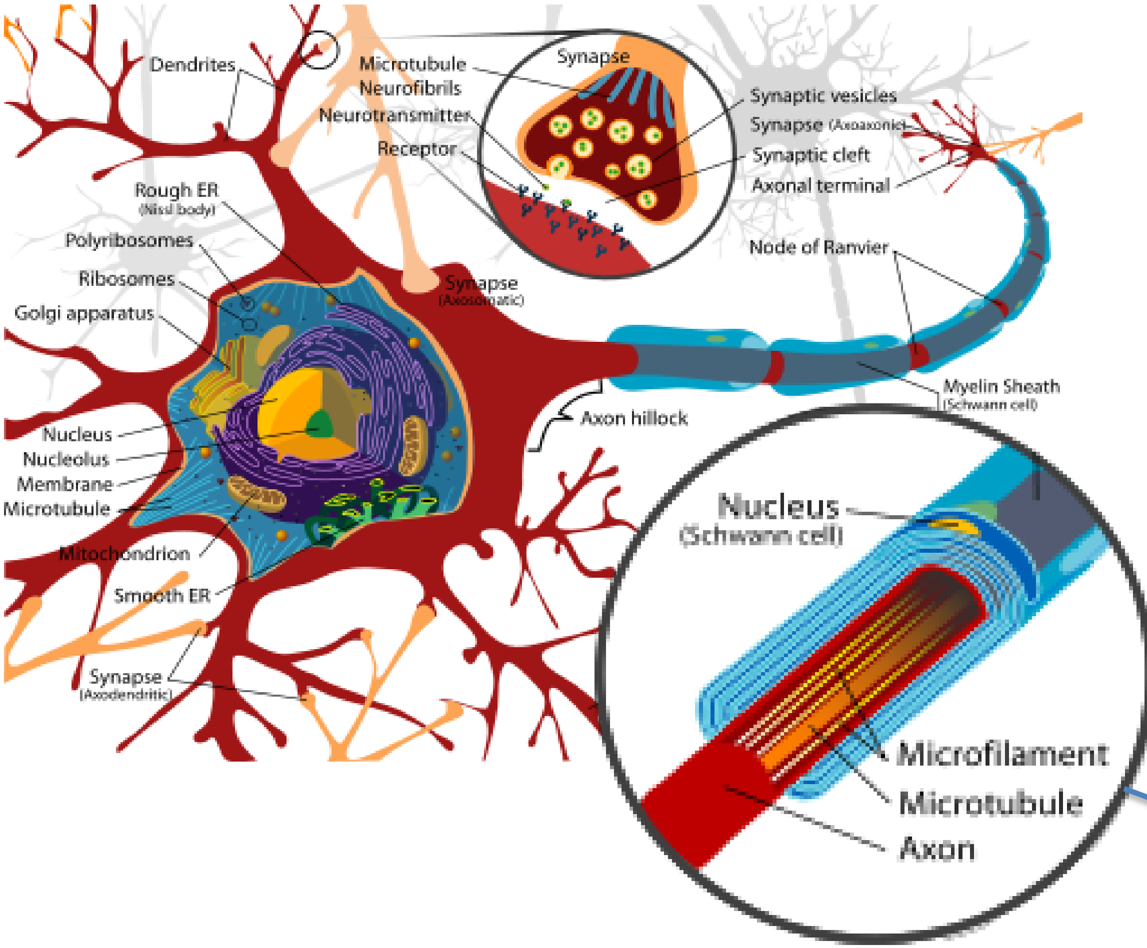

$\alpha = 0.1$



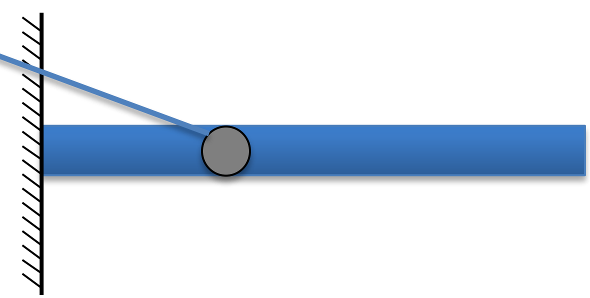



# Wave Guide Models & Neuron Cell Structure




Uniform rod model of an axon

Non-uniform, transversely isotropic rod model of an axon



Source: en.wikipedia.org/wiki/Neuron

# Summary and Future Directions

- Nonlinear viscoelastic models needed for explaining experimental response data collected with brain tissue
    - Response depends on loading characteristics: frequency and amplitude

- Reduced-order models have been developed to study the response of rigid structures supported by soft matter and skull-brain system. Wave propagation phenomena in homogeneous and non-homogeneous fiber structures have been studied and energy localization behavior has been observed.
    - Evidence for high strains in the response shown
    - Transient response of the nonlinear material is highly affected by the amplitude of the load, the frequency components of the load and the dissipation of the soft tissue. Therefore, accurate determination of the material properties of soft tissue is crucial for developing models that capture transient nonlinear effects
    - Better understanding of energy localization can help identify vulnerable areas

- Future work to focus on further developing the reduced-order models and understanding wave phenomena in skull-brain system in conjunction with other computational modeling efforts (NSWC-Indian Head) in three-dimensional cases and guiding experiments at UMD &UMB to obtain information on model response as well as soft tissue properties.